SAT Subject Test
Math Level II
Practice Test 5 (AT)
Solutions (from CB)
## TABLE A

Answers to the Subject Test in Mathematics Level 2, Form 3RBC2, and Percentage of Students Answering Each Question Correctly

<table>
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<tr>
<th>Question Number</th>
<th>Correct Answer</th>
<th>Right</th>
<th>Wrong</th>
<th>Percentage of Students Answering the Question Correctly*</th>
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*These percentages are based on an analysis of the answer sheets of a representative sample of 9,983 students who took the original form of this test in June 1995, and whose mean score was 649. They may be used as an indication of the relative difficulty of a particular question. Each percentage may also be used to predict the likelihood that a typical SAT Subject Test in Mathematics Level 2 candidate will answer that question correctly on this edition of the test.
The solutions presented here provide one method for solving each of the problems on this test. Other mathematically correct approaches are possible.

1. Choice (B) is the correct answer. Since \( 1 - \frac{1}{x} = 3 - \frac{3}{x} \), then \( \frac{2}{x} = 2 \). Solving for \( x \) gives \( x = 1 \). The value of \( 1 - \frac{1}{x} \) when \( x = 1 \) is equal to \( 1 - \frac{1}{1} = 0 \).

2. Choice (D) is the correct answer. Using the distributive property, \( a \left( \frac{1}{b} + \frac{1}{c} \right) = \frac{a}{b} + \frac{a}{c} \).
   To add these fractions, you need to find the least common denominator, which is \( bc \).
   Thus, \( \frac{a}{b} + \frac{a}{c} = \frac{ac}{bc} + \frac{ab}{bc} \), which is equivalent to choice (D).

3. Choice (D) is the correct answer. On the closed interval \([0, 2\pi]\), the minimum value of \( y = \sin x \) occurs when \( x = \frac{3\pi}{2} \). \( \sin \left( \frac{3\pi}{2} \right) = -1 \). Thus, the coordinates of \( P \) are \( \left( \frac{3\pi}{2}, -1 \right) \). Using a graphing calculator to see the graph of \( y = \sin x \) may be helpful in solving this problem.

4. Choice (A) is the correct answer. The set of all points in the plane the same distance from \( P \) and \( Q \) is the line that is perpendicular to \( PQ \) and bisects \( PQ \). Thus, the set of all points closer to \( P \) than \( Q \) is the region in the plane on the side of the line where \( P \) lies. The other choices do not include ALL such points that are closer to \( P \) than \( Q \).

5. Choice (C) is the correct answer. If \( \sqrt{6y} = 4.73 \), then \( 6y = 4.73^2 = 22.3729 \) and \( y = 3.729 = 3.73 \).

6. Choice (A) is the correct answer. The cosine of an angle is equal to \( \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} \). Thus, \( \cos \theta = \frac{x}{r} \), and \( r \cos \theta = r \left( \frac{x}{r} \right) = x \).
7. Choice (C) is the correct answer. \( f(10) = \sqrt{0.3(10)^2} - 10 = \sqrt{20} \approx 4.472 \) and \( g(\sqrt{20}) = \frac{\sqrt{20} + 1}{\sqrt{20} - 1} \approx 1.576 \approx 1.6. \)

8. Choice (C) is the correct answer. The equation given is equivalent to \( n^4 p^7 p^6 = 4n^2 p^7 p^6. \) This simplifies to \( n^4 = 4n^2. \) Dividing both sides by \( n^2, \) you get \( n = 4. \)

9. Choice (B) is the correct answer because there is not enough information given. If \( OA = AB, \) then \( \triangle OAB \) is an isosceles triangle. The slope of \( \overline{AB} \) can be found if the measure of \( \angle ABO \) is known or if the coordinates of \( A \) can be determined. In this problem, point \( A \) is not fixed vertically.

10. Choice (A) is the correct answer. Since \( \csc(2\theta) = \frac{1}{\sin(2\theta)}, \) \( \csc(2\theta) \sin(2\theta) = 1. \)

11. Choice (E) is the correct answer. Since \( |f(x)| \geq 0, \) the graph of \( y = |f(x)| \) consists of points \( (x, y), \) where \( y \geq 0. \) This eliminates choices (C) and (D). The graphs of \( y = f(x) \) and \( y = |f(x)| \) are identical where \( f(x) \geq 0. \) This eliminates choice (A). The portion of the graph of \( y = f(x) \) where \( f(x) < 0 \) must be reflected about the \( x \)-axis to produce the graph of \( y = |f(x)|. \) This eliminates choice (B) because it does not include the reflection of \( y = f(x) \) where \( f(x) < 0. \) Choice (E) is the complete graph of \( y = |f(x)|. \)

12. Choice (B) is the correct answer. If 3 and \(-2\) are zeros of \( p(x), \) then \((x - 3)(x + 2)\) are factors of \( p(x). \) \((x - 3)(x + 2) = x^2 - x - 6, \) which is also a factor of \( p(x). \) Choices (A), (C), (D), and (E) are incorrect. These choices result from sign errors in determining the factors that give the zeros of \( p(x) \) or sign errors in the multiplication of those factors.
13. Choice (C) is the correct answer. It is helpful to draw a figure to solve this problem. $x$ represents the distance from the kite to the ground, and $\sin 49^\circ = \frac{x}{100}$. Solving for $x$ gives $x = 75.471$, which is closest to choice (C). Choice (A) is incorrect. It results from using the incorrect equation $\sin 49^\circ = \frac{100}{x}$. Choice (B) is incorrect. It results from using $\tan 49^\circ$ instead of $\sin 49^\circ$. Choice (D) is incorrect. It results from using $\cos 49^\circ$ instead of $\sin 49^\circ$. Choice (E) is incorrect. It results from using the incorrect equation $\cos 49^\circ = \frac{100}{x}$ and then subtracting 100 from the solution to the equation.

14. Choice (A) is the correct answer, since $7x - 5 = 2$ when $x = 1$. None of the other choices satisfy $g(1) = 2$. One way to solve this problem is to first find the value of $x$ for which $f(x) = 11$. Since $3x + 5 = 11$, then $x = 2$. This implies that $g(1) = 2$, and we must determine which of the choices is equal to 2 when $x = 1$.

15. Choice (C) is the correct answer. You can use the Pythagorean theorem to find the length of $AB$. $(3)^2 + (1.5)^2 = x^2$ and $x = \sqrt{11.25} \approx 3.35$ cm. All sides of $ABCD$ have the same length, so its perimeter is $4x = 13.416 \approx 13.42$ cm. Choice (B) is incorrect. It is the area, in square centimeters, of $ABCD$. Choice (E) is incorrect. It is the perimeter if 11.25 cm is used as the length of a side.

16. Choice (B) is the correct answer. Since line $\ell$ is perpendicular to the $y$-axis and intersects the $y$-axis at $(0, 2)$, each point on line $\ell$ has the $y$-coordinate 2, and therefore, the equation of $\ell$ is $y = 2$.

17. Choice (B) is the correct answer. You can set up an equation to solve this problem.
Let $x$ be the weight of the new student, in pounds. The total weight of the 20 students is equal to $19(112) + x$. Since the mean weight is 111, it follows that $\frac{19(112) + x}{20} = 111$. Solving for $x$ gives 92 as the weight of the new student.
18. Choice (C) is the correct answer. For \(0 < x < \pi\) you can set your calculator in radian mode to find the value of \(x\), which is equal to \(\cos^{-1}(0.875) = 0.5054\). Keep this value in your calculator to evaluate \(\tan\left(\frac{x}{2}\right)\). \(\tan\left(\frac{x}{2}\right) = \tan\left(\frac{0.5054}{2}\right) = 0.2582\). Choice (D) is incorrect. It is equal to \(\frac{\tan(28.955^\circ)}{2}\), where \(28.955^\circ = \cos^{-1}(0.875)\) with the calculator set in degree mode. Choice (E) is incorrect. It is equal to \(\tan(28.955^\circ)\), where \(28.955^\circ = \cos^{-1}(0.875)\) with the calculator set in degree mode.

19. Choice (E) is the correct answer. You can set up an equation to solve this problem. Let \(\frac{x}{100}\) represent the percent of residents that voted "yes." 
\[
30,744 \left(\frac{x}{100}\right) + 20,496 \left(\frac{x}{100}\right) = 38,430.
\]
This simplifies to \(307.44x + 204.96x = 38,430\). Solving for \(x\) gives 75. Thus, 75% of the residents voted "yes," and 75% of 30,744 is 23,058. Choice (A) is incorrect. It is 25% of those who voted from Lyon County. Choice (C) is incorrect. It is 75% of those who voted from Saline County.

20. Choice (B) is the correct answer. In this problem, the function \(f\) maps a point \((x, y)\) in the plane to the point \((x + 2y, y)\). You are looking for all points at which the image has the same \(x\)-coordinate as the original point. If \(x = x + 2y\), then \(y\) must equal 0. Thus, you want all points \((x, y)\) such that \(y = 0\).

21. Choice (D) is the correct answer. Let \(x\) be the number to be added so that \(1 + x, 7 + x,\) and \(19 + x\) form a geometric progression. Then, \(\frac{19 + x}{7 + x} = \frac{7 + x}{1 + x}\). This equation simplifies to \(6x = 30\). Therefore, \(x = 5\).

22. Choice (D) is the correct answer. Since \(f(0) = 1, 1 = a(0) + b(0) + c\) and \(c = 1\). Since \(f(1) = 2, 2 = a(1) + b(1) + 1 = a + b + 1\). Thus, \(a + b = 1\).
23. Choice (C) is the correct answer. It is helpful to draw a figure to solve this problem. The largest angle is \( \angle B \), since it is opposite the longest side, The measure of \( \angle B \) can be found using the law of cosines.

\[
(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC) \cos \angle B
\]

\[
\cos \angle B = \frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)}
\]

\[
= \frac{49 - 36 - 36}{-2 \cdot 6 \cdot 6}
\]

Since \( \cos \angle B = \frac{23}{72} \), the measure of \( \angle B \) is \( \cos^{-1} \left( \frac{23}{72} \right) \approx 71.37^\circ \). Choice (B) is incorrect. It is the measure of \( \angle A \) and \( \angle C \).

24. Choice (E) is the correct answer. The cube root of any real number is a real number. Thus, the domain of \( f \) is all real numbers. Choices (B) and (C) are incorrect. They both incorrectly use \( \sqrt[3]{13} \approx 2.35 \). Choice (D) is incorrect. It incorrectly assumes that \(-x^2 + 13 \geq 0\).

25. Choice (E) is the correct answer. Since \( \tan x = \frac{\sin x}{\cos x} \), the equation can be rewritten as \( \cos x = \frac{\sin x}{\cos x} \), and \( \cos^2 x - \sin x = 0 \). Using the identity \( \sin^2 x + \cos^2 x = 1 \), the equation becomes \( 1 - \sin^2 x - \sin x = 0 \). This is a quadratic equation in \( \sin x \). Let \( y = \sin x \) and use the quadratic formula to solve \( y^2 + y - 1 = 0 \).

\[
y = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}
\]

\[
= \frac{-1 \pm \sqrt{5}}{2} = 0.6180, -1.6180
\]

\( \sin x = -1.6180 \) has no solution. Solving \( \sin x = 0.6180 \) gives \( x = \sin^{-1}(0.6180) \approx 0.67 \). Alternatively, you can use a graphing calculator to graph \( y = \cos x \) and \( y = \tan x \) on the interval \([0, \frac{\pi}{2}]\) and find the \( x \)-coordinate of the point of intersection.
26. Choice (D) is the correct answer. The width of each rectangle is 2. The heights of the rectangles are $3^0$, $3^2$, and $3^4$, respectively. The sum of the areas is $3^0(2) + 3^2(2) + 3^4(2) = 2 + 18 + 162 = 182$. Choice (B) is incorrect. It results from using the right endpoint instead of the left one for the heights of the rectangles ($3^2$, $3^4$, and $3^6$). Choice (C) is incorrect. It results from using the right endpoint for the heights of the rectangles and forgetting to multiply by 2. Choice (E) is incorrect. It results from forgetting to multiply by 2.

27. Choice (C) is the correct answer. For the function given, let $a = 600$ and $E(t) = 300$. Thus, $300 = 600e^{-t/1000}$, which simplifies to $\frac{1}{2} = e^{-t/1000}$. Taking the natural logarithm of both sides of the equation gives $\ln\left(\frac{1}{2}\right) = -\frac{t}{1000}$. Solving for $t$ yields $t = 693.147 \approx 693$. Alternatively, you can use a graphing calculator to graph $y = 300$ and $y = 600e^{-t/1000}$ and find the $x$-coordinate of the point of intersection.

28. Choice (D) is the correct answer. The lines $x = 0$ and $y = 1$ are asymptotes of the graph of $y = \frac{1+x}{x}$. The correct answer is I and III only. The line $y = 1$ is a horizontal asymptote, because as the value of $x$ increases without bound, the value of $y$ approaches 1. The line $x = 0$ is a vertical asymptote, because the value of $y$ is undefined when $x = 0$.

29. Choice (C) is the correct answer. To find an expression for $f(x)$, you must understand what $f(2x+1)$ means. If $f$ is evaluated at $2x+1$, the value is $2x-1$, which is equal to $(2x+1)-2$. So, the “input” value for $f$ has 2 subtracted from it to produce the “output” value of the function. Thus, if the original “input” value is $x$, the “output” value is $x-2$. Therefore, $f(x) = x - 2$.

30. Choice (D) is the correct answer. In order for the circle to be tangent to both the $x$-axis and $y$-axis, the center of the circle must be the same distance from both axes. Choices (A) and (C) are incorrect. They can be eliminated because they are each on a coordinate axis. Choices (B) and (E) are incorrect. They can be eliminated because each of these points is closer to one of the coordinate axes than the other. $(2, -2)$ is the answer because it is 2 units from both coordinate axes.
31. Choice (E) is the correct answer. To find the range of this piecewise-defined function, you must consider both parts. For \( x > 2 \), \( f(x) = x^{\frac{3}{2}} \). The range of the function is \( y > 2^{\frac{3}{2}} \), since the function is increasing for all \( x > 2 \). For \( x \leq 2 \), \( f(x) = 2x - 1 \). The range of this function is \( y \leq 3 \), since the function is decreasing as \( x \) is decreasing for \( x \leq 2 \). Combining \( y > 2^{\frac{3}{2}} \) and \( y \leq 3 \) gives all real numbers for the range. Choices (A) and (B) are incorrect. They result from considering only one part of the function. Choice (C) is incorrect. It results from incorrectly looking at the interval between the endpoints of the ranges of the respective parts.

32. Choice (C) is the correct answer. Since \( 2y - x^2 = 0 \), \( y = \frac{x^2}{2} \) for \( x \geq 0 \). Substituting that into the first equation yields \( 3x - 4\left(\frac{x^2}{2}\right) + 7 = 0 \). This simplifies to \( 3x - 2x^2 + 7 = 0 \) or \( 2x^2 - 3x - 7 = 0 \). Using the quadratic formula,
\[
x = \frac{3 \pm \sqrt{9 - 4(-2)(-7)}}{4}
\]
\[
= \frac{3 \pm \sqrt{65}}{4}
\]
\[
= 2.77, -1.27.
\]
Since \( x \geq 0 \), the answer is 2.77. Choice (A) is incorrect. It results from a sign error in solving for \( x \).

33. Choice (A) is the correct answer. The inverse of a logarithmic function with base \( a \) (\( f(x) = \log_a x \), where \( x > 0 \)) is an exponential function with base \( a \) (\( f^{-1}(x) = a^x \)). In this case, the inverse of \( f(x) = \log_2 x \) for \( x > 0 \) is \( f^{-1}(x) = 2^x \).

34. Choice (C) is the correct answer. Since \( x_0 = 0 \), \( x_1 = \sqrt{6} + x_0 = \sqrt{6} \approx 2.449 \), which is choice (A). Choice (A) is incorrect. \( x_2 = \sqrt{6} + x_1 = \sqrt{6} + \sqrt{6} \approx 2.907 \), which is choice (B). Choice (B) is incorrect. \( x_3 = \sqrt{6} + x_2 = \sqrt{6} + \sqrt{6} + \sqrt{6} = 2.984 \). Choice (D) is incorrect. It is \( x_4 \).
35. Choice (E) is the correct answer. Since the triangle is inscribed in a semicircle, it is a right triangle. The area of the triangle is equal to \( \frac{1}{2}ab \sin \theta \), where \( a \) and \( b \) represent adjacent sides and \( \theta \) is the included angle. In the figure, \( \cos \theta = \frac{y}{2} \) and \( y = 2 \cos \theta \). Thus, the area of the triangle is equal to \( \frac{1}{2}(2)(2 \cos \theta) \sin \theta = 2 \cos \theta \sin \theta \) which is equivalent to choice (E).

36. Choice (A) is the correct answer. Since the use of each thermometer is an independent event, the probability is equal to \((0.2)^4 = 0.0016\).

37. Choice (A) is the correct answer. Vectors \( \mathbf{a} \), \( \mathbf{b} \), and \( \mathbf{b} - \mathbf{a} \) can be represented as shown in the figure. Using the triangle inequality, \( 7 \leq \text{magnitude of } \mathbf{b} - \mathbf{a} \leq 17 \). Thus, the magnitude of \( \mathbf{b} - \mathbf{a} \) cannot be 5. The other choices are all possible magnitudes.

38. Choice (D) is the correct answer. Since \((6.31)^m = (3.02)^n\), \( \log(6.31)^m = \log(3.02)^n \) and \( m \log 6.31 = n \log 3.02 \). \( \frac{m}{n} = \frac{\log 3.02}{\log 6.31} = 0.60 \). Choice (A) is incorrect. It is equal to \( \log \left( \frac{3.02}{6.31} \right) \). Choice (C) is incorrect. It is equal to \( \frac{3.02}{6.31} \). Choice (E) is incorrect. It is equal to \( \frac{\log 6.31}{\log 3.02} \).

39. Choice (A) is the correct answer. \( \cos x \) and \( \arccos x \) are inverses of each other on the interval \( 0 \leq x \leq \frac{\pi}{2} \). If \( \arccos (\cos x) = 0 \), \( x \) could equal 0.
40. Choice (B) is the correct answer. You can use the given information to set up two equations. Let \( a_1 \) represent the first term of the arithmetic sequence, and let \( d \) represent the common difference.

\[
100 = a_{20} = a_1 + (20 - 1) d \\
250 = a_{40} = a_1 + (40 - 1) d
\]

Solving both of these for \( a_1 \) yields \( a_1 = 100 - 19d \) and \( a_1 = 250 - 39d \).

\[
100 - 19d = 250 - 39d \\
d = \frac{15}{2} = 7.5 \\
a_1 = 100 - 19(7.5) = -42.5
\]

Choice (A) is incorrect. It results from using \( a_{20} = a_1 + 20d \) and \( a_{40} = a_1 + 40d \). Choice (C) is incorrect. It results from thinking that the first term is \( \frac{100}{20} \). Choice (D) is incorrect. It results from a sign error.

41. Choice (B) is the correct answer. When \( n \) distinct planes intersect in a line, no two of the planes are parallel. So if another line \( \ell \) intersects one of these planes in a single point, it is parallel to at most one of the planes. Therefore, line \( \ell \) would intersect at least \( n - 1 \) planes. Thus, the least number of these \( n \) planes that line \( \ell \) intersects is \( n - 1 \).

42. Choice (D) is the correct answer. Since \( \sin \theta \) is odd, \( \sin(-\theta) = -\sin(\theta) \). Since \( \cos \theta \) is even, \( \cos(-\theta) = \cos(\theta) \). Thus, \( \sin \theta + \sin(-\theta) + \cos \theta + \cos(-\theta) = \sin \theta - \sin(\theta) + \cos \theta + \cos \theta = 2 \cos \theta \).

43. Choice (B) is the correct answer. Since \( n! = (n-1)!n \), then \( \frac{(n-1)!}{(n-1)!} = \frac{n-1}{n} \) and \( \frac{(n-1)n}{n!} = \frac{1}{n^2} \).
Choice (E) is the correct answer. It is helpful to draw a figure. This problem can be solved using similar triangles. Setting up the proportion $\frac{4}{6} = \frac{h-8}{h}$ results in

\[4h = 6(h - 8)\]
\[4h = 6h - 48\]
\[48 = 2h\]
\[h = 24\]

Choice (C) is incorrect. It is the height of the smaller cone whose base is the parallel cross section.

45. Choice (C) is the correct answer. An indirect proof begins with assuming the negative of the conclusion. The conclusion is "\(\sqrt{x}\) is NOT a rational number." The negative of this statement is "\(\sqrt{x}\) is a rational number."
46. It may be helpful to draw a graph of \( f \) and \( g \).

Choice (B) is the correct answer. The function \( g \) is given by \( g(x) = -(x+3)^2 + 1 \). Therefore, \( g(-1.6) = -(1.6+3)^2 + 1 = -0.96 \). Choice (A) is incorrect. It results from using \( g(x) = (x+3)^2 + 1 \). Choice (C) is incorrect. It is \( f(-1.6) + 1 \). Choice (D) is incorrect. It results from using \( g(x) = -(x+3)^2 \). Choice (E) is incorrect. It is \( f(-1.6) \).

47. Choice (A) is the correct answer. To determine the number of ways that 10 people can be divided into the two groups, find either \( \binom{10}{7} \) or \( \binom{10}{3} \), which are equivalent. Once the number of ways to form one of the groups is determined, there is only one way to form the other group. So, \( \binom{10}{7} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 7!} = 120 \).

48. Choice (A) is the correct answer. You need to examine each set separately. Consider I. There is no least positive rational number, so “the set of positive rational numbers” does not satisfy the desired condition. Consider II. \( \sqrt{2} \) is the smallest positive real number that satisfies \( r^2 \geq 2 \), but \( \sqrt{2} \) is irrational. Thus, there is no smallest positive rational number that satisfies the desired condition. Consider III. \( r > 2 \), but there is no smallest rational number that satisfies this condition. None of the three sets has an element that is less than any other element in the set.
49. Choice (D) is the correct answer. The standard form for the equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. $60x^2 + 30y^2 = 150$ can be rewritten as $\frac{60x^2}{150} + \frac{30y^2}{150} = \frac{150}{150}$, which is equivalent to $\frac{x^2}{2.5} + \frac{y^2}{5} = 1$. Because the denominator of the $y^2$ term is larger than the denominator of the $x^2$ term, the major axis of this ellipse is vertical. Since $b^2 = 5$, the vertices are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$. The length of the major axis is $2b = 2\sqrt{5} = 4.47$. Choice (B) is incorrect. It is $a^2$. Choice (C) is incorrect. It is $2a = 2\sqrt{2.5} \approx 3.16$, which is the length of the minor axis of this ellipse. Choice (E) is incorrect. It is $b^2$.

50. Choice (C) is the correct answer. You need to determine for which of the conditions $\frac{a-b}{ab} > 0$. It is helpful to examine each of the answer choices. Choice (A) is incorrect. $0 < a < b$. In this case, $a - b < 0$ and $ab > 0$, so the expression is NEGATIVE. Choice (B) is incorrect. $a < b < 0$. In this case, $a - b < 0$ and $ab > 0$, so the expression is NEGATIVE. In this case, $a - b > 0$ and $ab > 0$, so the expression is POSITIVE. You are looking for a positive result. Choice (D) is incorrect. $b < 0 < a$. In this case, $a - b > 0$ and $ab < 0$, so the expression is NEGATIVE.