3. **D** Take either the log or natural log of both sides of the equation to solve for $x$.

$$2^x = 5.$$  
$$\ln 2^x = \ln 5.$$  

Based on the properties of logarithms, the equation can be written as:

$$x \ln 2 = \ln 5.$$  
$$x = \frac{\ln 5}{\ln 2} = 2.321928.$$  
$$5^x = 5^{2.321928} = 41.97.$$  

4. **B** Square both sides to solve for $x$.

$$\sqrt[7]{x} = 6.24.$$  
$$7x = 6.24^2 = 38.9376.$$  
$$x = \frac{38.9376}{7} = 5.56.$$
5. **E** By definition, an ellipse is the set of all points \((x, y)\) in a plane the sum of whose distance from two distinct foci is constant. Answer **E** is the correct choice.

6. **C** Recall that cosine and secant are reciprocal functions.

\[
\cos(4\theta) \sec(4\theta) =
\]

\[
\cos(4\theta) \left( \frac{1}{\cos(4\theta)} \right) =
\]

\[
\frac{\cos(4\theta)}{\cos(4\theta)} = 1.
\]

7. **D** The graph of the line containing the point \((-5, 2)\) that is parallel to the y-axis and perpendicular to the x-axis is as follows:

![Graph of a line](image)

\(x = -5\) is the correct answer choice.

8. **A**

Distance:

\[
\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}
\]

\[
= \sqrt{(3 - 2)^2 + (4 - 1)^2 + (-1 - 3)^2}
\]

\[
= \sqrt{5^2 + 3^2 + (-4)^2}
\]

\[
= \sqrt{50} = 5 \sqrt{2}.
\]

9. **C**

\[
\sin \frac{\pi}{2} = 1.
\]

Because \(\cos 2\pi\) also equals 1, Answer **C** is the correct answer choice.

10. **C** If 1 and \(-4\) are both roots of a given polynomial, then \((x - 1)\) and \((x + 4)\) must be factors of the polynomial. The product of the two must also be a factor:

\[(x - 1)(x + 4) =
\]

\[x^2 + 3x - 4.
\]

11. **B** The sine function can be thought of as \(y = c + a \sin bx\) whose period is \(2\frac{\pi}{b}\). The period of \(y = \sin 2x\) is \(2\frac{\pi}{2} = \pi\), and the maximum value of the sine function is 1. This occurs at \(\frac{\pi}{4}\) on the interval \(0 \leq x < \pi\). The coordinates of the point where the maximum value occurs are, therefore, \(\left(\frac{\pi}{4}, 1\right)\).

12. **B** You may recall that in the polar coordinate system \(y = r \sin \theta\), so \(y\) is the correct answer choice. An alternate way to solve the problem is to use right triangle trigonometry to determine that \(\sin \theta = \frac{y}{r}\).

\[r \sin \theta = r \left(\frac{y}{r}\right) = y\]

13. **A**

\[f(g(x)) = -\frac{1}{3}\] \(g(x) = x\), so

\[4(g(6)) - 1 = -\frac{1}{3},
\]

\[4(g(6)) = \frac{2}{3},
\]

\[g(6) = \frac{2}{12} = \frac{1}{6}.
\]

If \(g(x) = \frac{1}{x}\), then \(g(6) = \frac{1}{6} \cdot g(x) = \frac{1}{x}\) is a possible solution.

14. **E**

Let \(h\) be the height of the flagpole.

\[\tan 42^\circ = \frac{h}{50},
\]

\[h = 50 \tan 42^\circ,
\]

\[h = 45 \text{ feet}.
\]
15. B
\[ f(12) = \sqrt{(12 - 4)} = \sqrt{8} = 2\sqrt{2}. \]
\[ g(f(12)) = g(2\sqrt{2})^3 + 1 = 8(\sqrt{2})^3 + 1 = 16\sqrt{2} + 1. \]

16. E The radicand in a cube root can be either positive or negative. This differs from the radicand in a square root. If the problem asked for the domain of \( f(x) = \sqrt[3]{4x^2 - 1} \), then \( 4x^2 - 1 \) would be have to be greater than or equal to zero. Here, there is no restriction on \( x \), so the domain of the function is the set of all real numbers.

17. E Set up an equation to represent the exponential decay, \( y = Ce^{-kt} \) where \( C \) is the initial amount of the substance, \( k \) is the rate of decay, and \( t \) is the time. It is given that the half-life of the substance is 9 years, meaning that half of the initial amount, 20 grams, will remain in 9 years. Use the half-life to solve for the rate of decay.
\[ 20 = 40e^{-10k} \]
\[ \frac{1}{2} = e^{-9k} \]
Now, take the natural log of both sides. Use the properties of logarithms and the fact that \( \ln e = 1 \) to solve for \( k \).
\[ \ln \frac{1}{2} = \ln e^{-9k} \]
\[ \ln \frac{1}{2} = -9k \ln e \]
\[ \ln \frac{1}{2} = -9k \]
\[ k = 0.077. \]
Use \( k \) to solve for \( y \) when \( t = 23.5 \) years.
\[ y = Ce^{-kt} \]
\[ y = 40e^{-0.077(23.5)} \]
\[ y = 6.55 \text{ grams}. \]

18. B The quadratic equation that has roots \( 3 + i \) and \( 3 - i \) is \( x^2 - 6x + 10 = 0 \)
\[ x = 3 + i \] or \( x = 3 - i \)
\[ x - 3 - i = 0 \] or \( x - 3 + i = 0 \)
\[ (x - 3)(x - 3 + i)(x - 3 - i) = 0 \]
\[ ((x - 3) - i)((x - 3) + i) = 0 \]
\[ [x - 3]^2 - i^2 = 0 \]
\[ x^2 - 6x + 9 - (-1) = 0 \text{ since } i^2 = -1 \]
\[ x^2 - 6x + 10 = 0. \]

19. B Factor the numerator and denominator. Then, simplify the expression and evaluate it when \( x = 5 \).
\[ f(x) = \frac{x^2 - 25}{x - 5} = \frac{(x - 5)(x + 5)}{x - 5} \]
\[ = x + 5, \]
when \( x = 5 \), \( x + 5 = 10 \).

20. A The figure depicts the graph of \( y = x^2 - 3 \). In the graph of \( y = f(x) \), \( y \) must be positive because it equals the absolute value of an expression. B and C can be eliminated as possible answer choices because they contain negative \( y \) values. Reflecting the portion of the graph of \( y = f(x) \) in which \( y < 0 \) over the \( x \)-axis results in the correct graph. Answer A is the correct answer choice.

21. C The center of the hyperbola is the midpoint of the segment connecting \((0, 4)\) and \((6, 4)\), so the center is \((3, 4)\). The distance from the center to each focus point is \(3\), so \( c = 3\). The distance from the center to each vertex is \(2\), so \( a = 2\). The hyperbola opens right and left because the transverse axis connecting \((1, 4)\) and \((5, 4)\) is horizontal.
\[ c^2 = a^2 + b^2, \]
\[ 3^2 = 2^2 + b^2, \]
\[ 5 = b^2. \]

The standard form of the equation of a hyperbola opening right and left is \[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]
where \((h, k)\) is the center. This results in:
\[ \frac{(x - 3)^2}{4} - \frac{(y - 4)^2}{5} = 1. \]
22. D Find the number of permutations of six letters taken six at a time.

\[ 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720. \]

23. A

\[ a_2 = a_1r^1 = 6 \text{ and } a_5 = a_1r^4 = \frac{81}{4}. \]

You can now write \( a_5 \) in terms of \( a_2 \).

\[ 6 \times r \times r \times r \times r = \frac{81}{4}. \]

\[ 6r^3 = \frac{81}{4}, \]

\[ r^3 = \frac{81}{24} = \frac{27}{8}. \]

\[ r = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}. \]

24. B Complete the square to get a perfect binomial squared plus a constant.

\[ x^2 - 5x + 1 = \]

\[ \left[ x^2 - 5x + \left(\frac{5}{2}\right)^2 \right] - \left(\frac{5}{2}\right)^2 + 1 = \]

\[ \left( x - \frac{5}{2} \right)^2 - \frac{25}{4} + \frac{4}{4} = \]

\[ \left( x - \frac{5}{2} \right)^2 - \frac{21}{4}, \text{ so } c = -\frac{21}{4}. \]

25. C Isolate the radical expression and square both sides to solve for \( x \).

\[ 3x - 5\sqrt{x} - 2 = 0. \]

\[ 3x - 2 = 5\sqrt{x}. \]

\[ (3x - 2)^2 = (5\sqrt{x})^2. \]

\[ 9x^2 - 12x + 4 = 25x. \]

\[ 9x^2 - 37x + 4 = 0. \]

\[ (9x - 1)(x - 4) = 0. \]

\[ x = \frac{1}{9} \text{ or } x = 4. \]

Remember to check for extraneous roots by substituting both solutions into the original equation. \( \frac{1}{9} \) is an extraneous root because \( 3\left(\frac{1}{9}\right) - 5\sqrt{\frac{1}{9}} - 2 \neq 0 \). The only solution is \( x = 4 \).

26. B Note that both 16 and 8 can be written in base 2.

\[ 16^{x-x} = 8^{x-x}, \]

\[ 2^{4(2-x)} = 2^{8(2-x)}. \]

Set the exponents equal to each other and solve for \( x \).

\[ 4(2 - x) = 3(8 - x). \]

\[ 8 - 4x = 24 - 3x. \]

\[ -16 = x. \]

27. D

\[ (f + g)(-2) = f(-2) + g(-2) \]

\[ = 4(-2) + 1 + (2)^2 - 3(-2) + 1. \]

\[ = -7 + 11 = 4. \]

28. B The binomial expansion of \((x + y)^5 = \)

\[ 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5. \]

Substitute \( x = 2x \) and \( y = -1 \) to get:

\[ 1(2x)^5 + 5(2x)^4(-1) + 10(2x)^3(-1)^2 + 10(2x)^2(-1)^3 \]

\[ + 5(2x)(-1)^4 + 1(-1)^5 \]

\[ = 32x^5 - 80x^4 + 80x^3 - 40x^2 + 20x - 1. \]

Of course, you could multiply \((2x - 1)(2x - 1)(2x - 1) \)

\((2x - 1)(2x - 1) \) and get the same product, but remembering the Binomial Theorem will save time.

29. E 7 different chords can be drawn from the first point. 6 additional chords can be drawn from the second point because the chords must be distinct, and one has already been drawn. 5 additional chords can be drawn from the third point because you cannot count the 2 already drawn. Continue this pattern to get:

\[ 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28 \text{ chords.} \]
30. D Recall that the diagonals of a rhombus are perpendicular, bisect each other, and bisect the vertex angles of the rhombus. Let $x$ and $y$ equal the measures of the angles as shown below.

$$\tan x = \frac{9}{12}$$

$$x = \tan^{-1} \frac{9}{12} = 36.87^\circ.$$  

$$\tan y = \frac{12}{9}$$

$$y = \tan^{-1} \frac{12}{9} = 53.13^\circ.$$  

The larger angle of the rhombus measures $2(53.13) = 106.26^\circ$.

31. C Because the three terms form an arithmetic sequence, there must be a common difference between consecutive terms. In other words, the difference between the 2nd and 1st terms must equal the difference between the 3rd and 2nd terms.

$$6n - 2 - 3n = 3n - 2.$$  

$$8n + 1 - (6n - 2) = 2n + 3.$$  

$$3n - 2 = 2n + 3.$$  

$$n = 5.$$  

The third term is $8(5) + 1 = 41$ and the difference between terms is $2(5) + 3 = 13$. The fourth term is, therefore, $41 + 13 = 54$.

32. D There are many ways to solve this problem. You could use either synthetic or long division to divide the polynomial by the given factors and determine which one results in a non-zero remainder. Alternatively, if $x - a$ is a factor of the polynomial, then $a$ is a zero.

$$f(-1) = (-1)^4 - 3(-1)^3 - 11(-1)^2 + 3(-1) + 10 = 0.$$  

$$f(1) = (1)^4 - 3(1)^3 - 11(1)^2 + 3(1) + 10 = 0.$$  

$$f(-2) = (-2)^4 - 3(-2)^3 - 11(-2)^2 + 3(-2) + 10 = 0.$$  

$$f(2) = (2)^4 - 3(2)^3 - 11(2)^2 + 3(2) + 10 = -36.$$  

$$f(5) = (5)^4 - 3(5)^3 - 11(5)^2 + 3(5) + 10 = 0.$$  

$f(2)$ results in a remainder of $-36$, so $x = 2$ is not a factor of the polynomial.

33. B

Because $f(0) = -1$, $a(0)^2 + b(0) + c = -1$.  

$-1 = a + b + c$.  

Because $f(1) = 3$, $a(1)^2 + b(1) + -1 = 3$.  

$a + b = 4$.  

Because $f(2) = 5$, $a(2)^2 + b(2) + -1 = 5$.  

$4a + 2b = 6$.

Set up a system to solve for $a$ and $b$.

$4a + 2b = 6$  

$-2(a + b) = -2(4)$  

$2a + 0b = -2$  

$a = -1$  

Now, solve for $b$ using $a + b = 4$.  

$-1 + b = 4$, so $b = 5$.

$$f(x) = ax^2 + bx + c = -x^2 + 5x - 1.$$  

34. A

$$(\sqrt{a^4 b^4}) = 10b^2.$$  

$$(a^2 b^2)^2 = 10b^2.$$  

$a^2 b^2 = 10b^2$.

Because $b 
eq 0$, $a^2 = 10$.

$a = \pm \sqrt{10}$.

Answer A is one possible solution for $a$. 
35. A 80% of 25 = 20 students passing with a grade of C or better. This means 20% of 25, or 5 students, are NOT passing with a grade of C or better. The probability of choosing two students that are NOT passing with a grade of C or better is:

\[
\frac{5}{25} \times \frac{4}{24} = \frac{1}{5} \times \frac{1}{6} = \frac{1}{30} = 0.03.
\]

36. \( f(4x - 8) = 2x - 2. \)

If \( f(x) = \frac{1}{2} x + 2, \) then;

\[
f(4x - 8) = \frac{1}{2}(4x - 8) + 2
\]

\[2x - 4 + 2 = 2x - 2.\]

Answer C is the correct choice.

37. B Given \( f(x) = 2x^3 + 6, \) interchange the \( x \) and \( y \) and solve for \( f^{-1}. \)

\[
y = 2x^3 + 6
\]

\[
x = 2y^3 + 6.
\]

\[x - 6 = 2y^3.\]

\[
y^3 = \frac{x - 6}{2}.
\]

\[
y = \left( \frac{x - 6}{2} \right)^{\frac{1}{3}} = f^{-1}(x).
\]

\[f^{-1}(10) = \left( \frac{-10 - 6}{2} \right)^{\frac{1}{3}} = -2.\]

38. D The circle given by the equation \((x + 3)^2 + (y - 3)^2 = 9\) has a center of \((-3, 3)\) and a radius of \(\sqrt{9} = 3\) units. Moving three units from the center, the circle intersects the \(y\)-axis at \((0, 3)\) and the \(x\)-axis at \((-3, 0)\).

39. B Because \(x^2 - y = 1, y = x^2 - 1.\) Substitute this value of \(y\) into the second equation to get:

\[11x - 2(x^2 - 1) = -4.\]

\[11x - 2x^2 + 2 = -4.
\]

\[0 = 2x^2 - 11x - 6.
\]

\[0 = (2x + 1)(x - 6).
\]

\[x = -\frac{1}{2} \text{ or } x = 6.
\]

Using the equation \(y = x^2 - 1, y = 6^2 - 1 = 35. x = -\frac{1}{2}\) does not result in a positive value of \(y\), so \(y = 35\) is the only solution.

40. D The volume of a right circular cone with radius \(r\) and height \(h\) is given in the Reference Information at the beginning of the test. \(V = \frac{1}{3} \pi r^2 h.\) Let \(d\) be the diameter of the cone. Then, \(\frac{d}{2}\) is the radius of the cone and \(2d\) is the height of the cone.

\[V = \frac{1}{3} \pi \left( \frac{d}{2} \right)^2 (2d) = 6.
\]

\[\pi \left( \frac{d^2}{4} \right) (2d) = 18.
\]

\[\pi d^3 = 36.
\]

\[d^3 = \frac{36}{\pi}.
\]

\[d = \left( \frac{36}{\pi} \right)^{\frac{1}{3}} \approx 2.25.
\]

41. D The function \(f_{n+1} = f_{n-1} - 2f_n\) is recursive. Because you are given the first two terms of the sequence, you can define the other terms using these.

\[f_1 = 2.
\]

\[f_2 = 6.
\]

\[f_3 = f_1 - 2f_2 = 2 - 2(6) = -10.
\]

\[f_4 = f_2 - 2f_3 = 6 - 2(-10) = 26.
\]
CHAPTER 3 / DIAGNOSTIC TEST

42. C Because \( x = 4t + 1, t = \frac{x - 1}{4} \). Substitute this value into the second equation to get:

\[
y = -3 + 2\left(\frac{x - 1}{4}\right).
\]

\[
2y = -6 + x - 1.
\]

\[
y = \frac{1}{2}x - \frac{7}{2}.
\]

The \( y \)-intercept of the resulting line is \(-\frac{7}{2}\).

43. E

Let \( s \) = the sum of the ages of the 21 students.

\[
\frac{s}{21} = 16.20.
\]

\[
s = 340.2.
\]

Let \( x \) = the age of the new student.

\[
\frac{340.2 + x}{22} = 16.27.
\]

\[
340.2 + x = 357.94.
\]

\[
x = 17.7 \text{ years}.
\]

44. B Choosing 8 people out of the twelve results in the following:

\[
\binom{12}{8} = \frac{12!}{8!(12 - 8)!} = \frac{12!}{8!4!} = \frac{9 \times 10 \times 11 \times 12}{1 \times 2 \times 3 \times 4} = 495.
\]

Note that once the 8 members are chosen, the remaining 4 people are automatically placed in the second group. There are no additional groupings to determine.

45. C If \( \sec \theta = \frac{5}{3} \), then \( \cos \theta = \frac{3}{5} \). Picture a 3-4-5 right triangle in quadrant I to determine that the value of \( \sin \theta = \frac{4}{5} \).

Recall the double angle formula for sine:

\[
\sin 2\theta = 2 \sin \theta \cos \theta.
\]

\[
\sin 2\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25} = 0.96.
\]

46. A Translating \( f(x) = |x| \) 5 units down and 1 unit left results in:

\( g(x) = |x + 1| - 5 \).

Now reflect the function over the \( x \)-axis to get:

\( g(x) = -(x + 1) - 5 \).

\( g(-4) = -(1-4+11-5) = -(1-3-5) = -(2) = 2 \).

47. E Because each score is decreased by 2, the mean also decreases by 2.

Recall that the standard deviation is given by the formula:

\[
\sigma = \sqrt{\frac{\text{sum of the squares of the deviations from the mean}}{\text{number of terms in the data set}}}.
\]

A deviation is the difference between a data value and the mean of the data set. When the scores are decreased by 2, the difference of each of the terms from the mean is unchanged, however, so the standard deviation remains unchanged. I and IV are true statements.
48. **D** The largest angle of a triangle is opposite its longest side. Let \( C \) be the triangle's largest angle. The side opposite \( \angle C \) is side \( AB \), and it measures 16 inches.

Recall that the Law of Cosines states:
\[
c^2 = a^2 + b^2 - 2ab \cos C,
\]
where \( a \), \( b \), and \( c \) are the lengths of the sides of the triangle.

\[
16^2 = 4^2 + 13^2 - 2(4)(13)\cos C. \\
16^2 = 16 + 169 - 104 \cos C. \\
71 = -104 \cos C. \\
\cos C = \frac{-71}{104}. \\
C = \cos^{-1}\left(\frac{-71}{104}\right) = 133.1^\circ.
\]

49. **C** Recall that the absolute value of a complex number is given by: \( |a + bi| = \sqrt{a^2 + b^2} \).

\[
|4 + 2i| = \sqrt{(4^2 + 2^2)} = \sqrt{20} = 2\sqrt{5}.
\]

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**PART I / ABOUT THE SAT MATH LEVEL 2 TEST**

50. **E** Using right triangle trigonometric ratios:

\[
\cos B = \frac{a}{c}.
\]

\[
\sin A = \frac{a}{c}.
\]

\[
\sec A = \frac{1}{\cos A} = \frac{c}{b}.
\]

\[
\cos B \sin A = \frac{\left(\frac{a}{c}\right)\left(\frac{a}{c}\right)}{\left(\frac{c}{b}\right)}.
\]

Multiply the numerator and denominator by the \( \text{LCD, } c^2b \), to get:

\[
\frac{a^2b}{c^2}.
\]