Name: Practice Problems for Probability

1. You toss a coin 4 times.
   (a) How many different equally likely outcomes are possible? \(2^4 = 16\)
   (b) Find the probability of obtaining no heads. \(P(\text{TTTT}) = \frac{1}{16}\)
   (c) Find the probability of obtaining at least one head. \(1 - P(\text{No Heads}) = 1 - \frac{1}{16} = \frac{15}{16}\)
   (d) Find the probability of obtaining exactly one head. \(P(\text{HTTH, THHT, THTH, TTHT}) = \frac{4}{16} = \frac{1}{4}\)
   (e) What are the odds of getting exactly one head? \(4:12 \text{ or } 1:3\)

2. A pile of six cards consists of 3 red cards, 2 blue cards, and 1 green card.
   (a) Assume you drew the green card, and then you drew a second card without replacing the green card back into the pile.
      What is the probability that the second card is: red? \(\frac{3}{5}\)
      blue? \(\frac{2}{5}\)
      green? \(0\)
   (b) Assume you drew the green card, and then replaced it back into the pile before you drew a second card. What is the probability that the second card is: red? \(\frac{1}{6}\)
      blue? \(\frac{2}{6}\)
      green? \(0\)

3. Two computers are used to control a space vehicle. The second computer shadows the first computer and is ready to take control if the first computer malfunctions. Suppose the probability of a computer failing on a space mission is 0.01%. Assume that computer failure is independent of another computer malfunctioning. What is the probability that the space mission will have correct computer support? \(1 - 0.0001^2 = 0.999999999\)
4. Two dice (six-sided number cubes) are rolled.

(a) What is the probability that the numbers are the same? \( P(\text{both numbers are the same}) = \frac{6}{36} = \frac{1}{6} \)

(b) What is the probability that the numbers are the same given that the sum of the numbers is 6?

\[ \frac{1}{5} \]
\[ \begin{align*}
 & (1, 5) \quad (2, 4) \quad (3, 3) \quad (4, 2) \quad (5, 1)
\end{align*} \]

(c) Are the events "both numbers are the same" and "their sum = 6" independent events? No

Explain with calculations.

\[ \frac{P(\text{both \# same})}{P(\text{sum} = 6)} \neq \frac{P(\text{sum} = 6 \mid \text{both \# same})}{P(\text{sum} = 6)} \]

\[ \frac{\frac{1}{6}}{\frac{5}{36}} \neq \frac{\frac{1}{6}}{\frac{5}{36}} \]

5. Assume the general population is 52% female, 48% male.

Good approximation

(a) What is the probability of drawing an all-female six-person jury?

\[ \binom{6}{6} (0.52)^6 (0.48)^0 \approx 0.0198 \]

(b) What is the probability of drawing at least one male on the jury?

\[ 1 - 0.0198 \approx 0.980 \]

6. Assume you randomly guess the answers to ten true-false questions on a quiz.

What is the probability that you will get at least 70% correct?

\[ \binom{10}{7} (0.5)^7 (0.5)^3 + \binom{10}{8} (0.5)^8 (0.5)^2 + \binom{10}{9} (0.5)^9 (0.5) + \binom{10}{10} (0.5)^{10} \approx 0.172 \]

7. Three cards are drawn from a deck without replacement.

(a) What is the probability of drawing all clubs?

\[ \frac{13 \cdot 3}{52 \cdot 3} = \frac{11}{850} \]

(b) What is the probability of drawing no clubs?

\[ \frac{39 \cdot 3}{52 \cdot 3} = \frac{703}{1700} \]

(c) What is the probability of drawing exactly 2 clubs?

\[ \frac{13 \cdot 2 \cdot 39 \cdot 1}{52 \cdot 3} = \frac{117}{850} \]

8. Four balls are chosen at random from a bag containing 5 black, 4 white, and 3 green balls.

(a) Find the probability of selecting 2 black and 2 white balls.

\[ \frac{4 \cdot 2 \cdot 5 \cdot 2}{12 \cdot 4} = \frac{60}{495} = \frac{4}{33} \approx 0.121 \]
9. Quality control has determined that each shipment of 200 batteries contains 2.5% defective ones.
Assume you purchase 5 batteries from a shipment.
\[
\frac{195}{200} \approx 0.975 \approx 0.880
\]
(a) What is the probability that all 5 are good batteries?
(b) What is the probability that at least one of the batteries is defective? \(1 - 0.880 \approx 0.120\)

10. Assume the general population is 52% female and 48% male.
Assume 60% of females live until at least age 75,
while only 42% of males live as long.
Determine the probability that a 75 year old person is female.
\[
P(F|75^+) = \frac{P(F \cap 75^+)}{P(75^+)} = \frac{0.312}{(0.312 + 0.208)} \approx 0.607
\]

11. A circular dart board is inscribed within a square having a diagonal length of 6 inches. If a dart is thrown blindly and it lands inside the square, what is the probability that the dart does not hit the circular dart board? Answer to the nearest thousandth. (4 points)
\[
36 - 9\pi \approx 0.215
\]
\[
(3\sqrt{2})^2 = 9 \times 2 = 18
\]
\[
A - A_0 = \pi \left(\frac{3\sqrt{2}}{2}\right)^2 = \frac{9\pi}{4}
\]
\[
A = \frac{9\pi}{2}
\]
\[
\frac{18 - \frac{9\pi}{2}}{18} \approx 0.214602
\]
12. Twenty-five people claimed to have witnessed a robbery. The number of witnesses describing the robber's height and weight were as follows: (4 points)

<table>
<thead>
<tr>
<th>Weight Range</th>
<th>5'8&quot; - 5'10&quot;</th>
<th>5'10&quot; - 6'0&quot;</th>
<th>Over 6'</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 - 180 lb.</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>180 - 200 lb.</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Over 200 lb.</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Find the probability that the robber was either over 6 feet tall or weighed from 180 to 200 pounds.

\[
\frac{12}{25} + \frac{8}{25} - \frac{5}{25} = \frac{15}{25} = \frac{3}{5} \approx 0.6
\]

13. Each shot that Peter takes in a basketball game has a 60% chance of going in the basket (scoring). If Peter takes eight shots in the game, what is the probability that at least 6 of his shots will go in the basket and score points? (4 points)

\[
\binom{8}{6}(0.6)^6(0.4)^2 + \binom{8}{7}(0.6)^7(0.4)^1 + \binom{8}{8}(0.6)^8(0.4)^0 = 0.315
\]

14. Jonathan represents that 97% of his red wine ferments properly. If you buy a case of wine (twelve bottles) at Jonathan's Redwine Vineyards, what is your best approximate probability that: (4 pts) \[0.693842\]

(a) all of the bottles are good wine \[12C_{12}(0.97)^{12}(0.03)^0 \approx 0.694\]

(b) at least one of the bottles is bad \[1 - 12C_{12}(0.97)^{12} \approx 0.306\]

15. In a certain lottery, five numbers are chosen without repetition from the integral numbers 1 through 20. If you correctly choose all five numbers in any order, you win $100,000.00. If you correctly pick 4 out of 5, or 3 out of 5, you win $1,000.00 or $100.00, respectively. If you purchase a $10.00 lottery ticket, what is your expected gain or loss on the lottery ticket? (5 points)

\[
\frac{5 \times 5}{20 \times 19} + \frac{5 \times 4 \times 15 \times 14 \times 13}{20 \times 19 \times 18 \times 17 \times 16} - \frac{5 \times 4 \times 3 \times 2 \times 1 \times 10}{20 \times 19 \times 18 \times 17 \times 16} = \frac{6.45 + 4.837461 + 6.772466 - 10}{1000000} = \frac{1}{1000000} = 8.06
\]

\[\times 1000000 = 8.06\]