16.5 Conditional Probability

\[ P(R) = \left(\frac{1}{2}\right) \left(\frac{\frac{2}{5}}{\frac{5}{5}}\right) + \left(\frac{1}{2}\right) \left(\frac{\frac{4}{5}}{\frac{5}{5}}\right) = \frac{1}{5} + \frac{2}{5} = \frac{3}{5} = 0.6 \]

\[ P(R_A) = \frac{\frac{5}{5}}{\frac{5}{5}} = \frac{1}{3} \text{ or } 0.3 \]

\[ P(R) = \left(\frac{1}{3}\right) \left(\frac{\frac{1}{3}}{\frac{1}{3}}\right) + \left(\frac{1}{3}\right) \left(\frac{\frac{2}{3}}{\frac{2}{3}}\right) + \left(\frac{1}{3}\right) \left(\frac{\frac{2}{3}}{\frac{2}{3}}\right) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{11}{24} \approx 0.458 \]

7) \[ P(\text{Defect}) = (0.6)(0.05) + (0.4)(0.12) = 0.03 + 0.008 = 0.038 \text{ or } 3.8\% \]

\[ P(\text{Defect}) = \frac{0.03}{0.038} \approx 0.789 \]

8) \[ P(\text{Accept}) = (0.95)(1) + (0.05)(0.08) = 0.95 + 0.004 = 0.954 \text{ or } 95.4\% \]

\[ P(\text{Accept and Defect}) = \frac{0.004}{0.954} \approx 0.00419 \]

9) \( a) \)
\[ P(\text{Accident}) = (0.22)(0.11) + (0.43)(0.03) + (0.02)(0.97) + (0.98) = 0.0242 + 0.0129 + 0.007 = 0.0441 \approx 4.41\% \]

\( b) \)
\[ P(\text{Accident and No Acc}) = \frac{0.0242}{0.0441} \approx 0.549 \]
10) \[
P(\text{No Driver's Ed} \& \text{Accident}) \div P(\text{Accident}) = \frac{(0.75)(0.13)}{[(0.75)(0.13) + (0.25)(0.05)]} = \frac{0.0975}{0.11} \approx 0.886
\]

11) \[
P(+ \& \text{disease}) = \frac{P (+)}{P (+) + 0.30} = \frac{(0.6)(\frac{58}{60})}{(0.6)(\frac{58}{60}) + (0.4)(\frac{40}{60})} \approx 0.853
\]

\[
P(- \& \text{No Disease}) = \frac{(0.4)(0.75)}{[(0.6)(\frac{10}{40}) + (0.4)(0.75)]} = \frac{0.3}{0.32} = 0.9375
\]
Conditional Probability

\[ P(D) = 0.85, \quad P(\overline{D}) = 0.15 \]

- Disease: \[ P(+ | D) = 0.99, \quad P(- | D) = 0.01 \]
- No Disease: \[ P(+ | \overline{D}) = 0.015, \quad P(- | \overline{D}) = 0.985 \]

(a) (1) False negative result: \[ 0.0085 \]
   (2) False positive result: \[ 0.00225 \]

(b) (1) Diagnostic test gives correct result: \[ (0.8415 + 0.14775) = 0.98925 \]
   (2) Person with positive test has the disease: \[ \frac{(0.8415)}{(0.8415) + (0.00225)} = 0.9973 \]
   (3) Person with negative test does not have the disease: \[ \frac{(0.14775)}{(0.14775) + (0.0085)} = 0.9456 \]
§ 16.5 Conditional Probability

15) 

\[ \frac{1}{2} \quad \frac{1}{2} \]

Fair pair of dice \hspace{1cm} Weighted pair of dice

\[ \frac{1}{6} \quad \frac{5}{6} \]

First toss \hspace{1cm} Second toss

\[ \frac{1}{6} \quad \frac{5}{6} \]

Six \hspace{1cm} No Six

\[ \frac{1}{6} \quad \frac{5}{6} \]

Six \hspace{1cm} No Six

\[ \frac{1}{4} \quad \frac{3}{4} \]

Six \hspace{1cm} No Six

\[ \frac{1}{4} \quad \frac{3}{4} \]

Six \hspace{1cm} No Six

Probability of 2 sixes with fair dice: \( \left( \frac{1}{2} \right) \left( \frac{1}{6} \right) \left( \frac{1}{6} \right) = \frac{1}{72} \)

Probability of 2 sixes with weighted dice: \( \left( \frac{1}{2} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) = \frac{1}{32} \)

Probability that 2 sixes came from weighted dice:

\[ \frac{\frac{1}{32}}{\frac{1}{32} + \frac{1}{72}} = \frac{9}{13} \text{ or } 0.692 \]