Using the binomial theorem with combination coefficients to calculate the probability of an outcome of an independent binomial event.

Events are independent if the outcome of each event is not related to the outcome(s) of previous events. An independent event is equivalent to “with replacement”. Each flip of a coin is an independent event; the outcome of each flip is unrelated to the result(s) of previous flip(s) of the coin.

A binomial event or experiment has the same two outcomes as the only outcomes for each successive event or trial or experiment. Examples of binomial outcomes are: success or failure, win or loss, heads or tails, right or left, off or on, not defective or defective for a product trial. The binomial theorem would not apply to rolling a die where there are six possible outcomes.

The probability of a binomial event can be calculated as follows:
Total # trials \( C \) total # successes (success rate as a decimal \( ^ \) # of successes) (failure rate \( ^ \) # of failures)

For example: An even-weighted coin is flipped four times.

What is the probability of getting three heads? \( 4 C 3 (0.5 ^ 3) (0.5 ^ 1) = 0.25 (\frac{1}{4}) \) or 25%
What is the probability of getting two heads? \( 4 C 2 (0.5 ^ 2) (0.5 ^ 2) = 0.375 (\frac{3}{8}) \) or 37.5%

The binomial theorem can also be used to approximate the probability of an outcome of a non-independent binomial event. The accuracy of the approximation improves as the sample size increases.

For example: Ryan makes 70% of his free throws in basketball games.

(a) What is the probability that Ryan will make two free throws in a row at the end of the game?
\( 2 C 2 (0.7 ^ 2) (0.3 ^ 0) = 0.49 \) (the probability of Ryan making two consecutive free throws) or 49%

(b) What is the probability that Ryan misses both free throws? \( 2 C 0 (0.7 ^ 0) (0.3 ^ 2) = 0.09 \) or 9%

(c) What is the probability that Ryan will make at least one of the two free throws? \( 1 - 0.09 = 0.91 \) or 91%

This is a binomial event (each free throw is either made or missed), but successive free throws are not independent events. If the first free throw is made, then Ryan’s 70% success rate increases; if the first free throw is missed, Ryan’s success rate sinks below 70%.

If Ryan’s original success rate of 70% is based on many previously taken free throw shots, then making or missing the first free throw will have a negligible effect on Ryan’s 70% success rate. The binomial theorem will be an accurate predictor of Ryan making or missing two successive free throws.
For example: A computer manufacturing company produces 8 defective computers out of every 1000 computers that it manufactures.

(a) What is the probability, that if I purchase 4 computers, none of the computers will be defective?

\[ 4 \binom{4}{0} (\frac{992}{1000})^4 (\frac{8}{1000})^0 = 0.968 \text{ or } 96.8\% \]

(b) What is the probability, that if I purchase 4 computers, exactly one will be defective?

\[ 4 \binom{4}{3} (\frac{992}{1000})^3 (\frac{8}{1000})^1 = 0.031 \text{ or } 3.1\% \]

(c) What is the probability, that if I purchase 4 computers, at least one of the computers will be defective?

\[ 1 - 0.968 = 0.032 \text{ or } 3.2\% \]

This is a binomial event (each computer is either defective or not defective), but the purchase of multiple computers are not independent events. It is equivalent to picking four successive computers without replacement. However, since the defect rate is based on a sampling of 1000 computers, a relatively large sampling, the binomial theorem should provide a relatively accurate approximation of the probability of purchasing defective or non-defective computers.