Probability

Probability is the likelihood that an event or outcome will happen. Probability is measured from 0 to 1.

A probability of 0 means there is no chance of the event happening; it never happens.

- E.g. the probability of rolling a 7 with one die

A probability of 1 means that the event always occurs; it is certain to happen; a certainty.

- E.g. the probability of rolling an integer with a die

A probability of 0.5 (or \(\frac{1}{2}\)) means the event is as likely to happen as not to happen.

- E.g. the probability of getting a heads on the flip of a coin; or getting a tails on the coin flip

2 types of probability: empirical (or experimental) v. theoretical.

- Empirical probability is based on actual events.
  - a limited number of actual trials

- Theoretical probability is based on an unlimited (infinite) number of random trials.

E.g. you flip a coin 10 times: getting 6 heads and 4 tails. The experimental probability of heads is 0.6 and the experimental probability of tails is 0.4.

If an evenly balanced (fair) coin is flipped an infinite number of times, the number of heads and tails will be equal.

The theoretical probability of heads is 0.5 (\(\frac{1}{2}\)) and the theoretical probability of tails is 0.5 (\(\frac{1}{2}\)).

Sample space: \{listing of all possible outcomes\} only single events are listed.

- each possible outcome is listed only once
- outcomes listed must be mutually exclusive (no overlap)
- outcomes listed must cover all possible outcomes.

- With empirical probability, the listed outcomes do not have to be equally likely to happen.
- With theoretical probability, the outcomes listed in the sample space must be equally likely.
- E.g. the sample space for rolling a die is \{1, 2, 3, 4, 5, 6\}

Empirical or experimental probability is calculated as follows:

\[
\frac{\text{# of successes}}{\text{# of times experiment is done (# tries)}}
\]

Theoretical probability is calculated:

\[
\frac{\# \text{ of favorable outcomes}}{\# \text{ of possible outcomes}}
\]

Probability of either of 2 events:

\[P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)\]

- If A and B are mutually exclusive events, then \[P(A \text{ or } B) = P(A) + P(B)\]

\[P(A \text{ or } \overline{A}) = 1\]

\[P(A \text{ and } \overline{A}) = 0\]

Complement principle: \[P(A) = 1 - P(\overline{A})\]

useful for calculating the probability of the occurrence of at least one ... \[1 - P(\text{none})\]

Geometric probability: \[\text{Target area} \div \text{Total area}\]

Odds: the chances of winning (or losing) a sporting event.

- E.g. If the odds for a game are 5:3 in favor of a team, then there is a \(\frac{5}{8}\) probability of winning (and a \(\frac{3}{8}\) probability of losing) the game.
Probability of either of two events: \( P (A \text{ or } B) \)

If \( A \) and \( B \) are mutually exclusive events, then \( P (A \text{ or } B) = P (A) + P (B) \)

\[ P (1 \text{ or Prime Number}) = P (1) + P (\text{Prime Number}) = \frac{1}{6} + \frac{3}{6} = \frac{2}{3} \]

If \( A \) and \( B \) are not mutually exclusive events, then \( P (A \text{ or } B) = P (A) + P (B) - P (A \text{ and } B) \)

\[ P (\text{Even or Prime Number}) = P (\text{Even}) + P (\text{Prime Number}) - P (\text{Even} \& \text{Prime}) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \]

Probability of both of two events: \( P (A \text{ and } B) \)

If \( A \) and \( B \) are independent events, then \( P (A \text{ and } B) = P (A) \cdot P (B) \)

\[ P (\text{Prime Number} \text{ and Prime Number}) = P (\text{Prime}) \cdot P (\text{Prime}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36} = \frac{1}{4} \]

If \( A \) and \( B \) are not independent events, but dependent events, then \( P (A \text{ and } B) = P (A) \cdot P (B | A) \)

(Probability of \( A \) \( \cdot \) (Recalculated) Probability of \( B \) given that \( A \) has already occurred)

\[ P (\text{Ace and Ace}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.00452 \]

\[ P (5 \text{ Diamonds}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{154,440}{311,875,200} = \frac{33}{66,640} \approx 0.000495 \]