What You'll Learn
- Recognizing periodic graphs and their features

...And Why
To make predictions about cyclic events such as sound waves

What You'll Need
- graph paper

4. Write 2 equations (one using sine, one cosine) to represent height v. time

\[ h(t) = -20 \cos \frac{\pi}{18}(t-4) + 25 \]
\[ h(t) = -20 \sin \frac{\pi}{18}(t-9) + 25 \]

Work with a partner.

You and a friend are the last people seated on a Ferris wheel. Once the ride begins, the wheel moves at a constant speed. It takes 36 seconds to complete one revolution.

1. a. When the ride starts, how high above the ground are you? 5 ft
   b. At what height are you after 9 s? after 18 s? after 27 s? 25, 45, 25
   c. At what height are you after 126 s? How many revolutions have you made? 45 ft, 3.5 rev.
   d. Predict where you will be after 3 min. 5 ft.

2. Sketch a graph showing the relationship between your height above the ground and the time since the ride began. Use 0 ≤ t ≤ 144 for the domain, where t = 0 is the time when the ride started.

3. Critical Thinking: How far have you traveled after one revolution of the wheel? after 144 s? 40π ≈ 125.664 ft., 502.655 ft.
The Frequency of a Sine Wave

Note that the graph of \( y = \cos x \) in Example 1 completes two cycles for every one completed by \( y = 5 \cos \left( \frac{x}{2} \right) \). We say that \( y = \cos x \) has twice the frequency of \( y = 5 \cos \left( \frac{x}{2} \right) \).

In general, the frequency of a periodic function is the reciprocal of the period, and represents the number of cycles the curve completes per unit of the independent variable. Thus, the frequency of the function \( y = \cos x \) is \( \frac{1}{2\pi} \), and the frequency of the function \( y = 5 \cos \left( \frac{x}{2} \right) \) is \( \frac{1}{4\pi} \). When a sine wave represents sound, doubling the frequency results in a pitch one octave higher. So the graph of \( y = \cos x \) represents a sound with pitch one octave higher than the pitch of \( y = 5 \cos \left( \frac{x}{2} \right) \).

**Example 3**

Suppose a tuning fork vibrates with a frequency of 440 cycles per second. If the vibration displaces air molecules by a maximum of 0.3 mm, give a possible equation for the sound wave that is produced.

\[ y = 0.3 \sin 880 \pi x \]

10. Suppose one tone has a frequency of 440 cycles per second, and another tone has a frequency of 880 cycles per second.
   a. Which has the higher pitch?
   b. How much higher is that pitch?

11. Consider a tuning fork vibrating at 512 cycles per second and displacing air molecules by a maximum of 0.14 mm. Give a possible equation for the sound wave that is produced.

\[ y = 0.14 \sin 1624 \pi x \]

12. **Multiple choice.** A sound wave whose parent graph is \( y = \sin x \) has five times the frequency and is four times as loud as the parent. What is a possible equation for this sound wave?
   (a) \( y = 5 \sin 4x \)  
   (b) \( y = 4 \sin 5x \)  
   (c) \( y = 4 \sin \frac{1}{5}x \)  
   (d) \( y = \frac{4}{5} \sin \frac{x}{5} \)