What You’ll Learn
- Recognizing periodic graphs and their features

...And Why
To make predictions about cyclic events such as sound waves

What You’ll Need
- graph paper

4. Write 2 equations (one using sine, one cosine) to represent height v. time

Work Together

You and a friend are the last people seated on a Ferris wheel. Once the ride begins, the wheel moves at a constant speed. It takes 36 seconds to complete one revolution.

1. a. When the ride starts, how high above the ground are you?
   b. At what height are you after 9 s? after 18 s? after 27 s?
   c. At what height are you after 126 s? How many revolutions have you made?
   d. Predict where you will be after 3 min.

2. Sketch a graph showing the relationship between your height above the ground and the time since the ride began. Use $0 \leq t \leq 144$ for the domain, where $t = 0$ is the time when the ride started.

3. Critical Thinking  How far have you traveled after one revolution of the wheel? after 144 s?
The Frequency of a Sine Wave

Note that the graph of \( y = \cos x \) in Example 1 completes two cycles for every one completed by \( y = 5 \cos \left( \frac{x}{2} \right) \). We say that \( y = \cos x \) has twice the frequency of \( y = 5 \cos \left( \frac{x}{2} \right) \).

In general, the frequency of a periodic function is the reciprocal of the period, and represents the number of cycles the curve completes per unit of the independent variable. Thus, the frequency of the function \( y = \cos x \) is \( \frac{1}{2\pi} \); and the frequency of the function \( y = 5 \cos \left( \frac{x}{2} \right) \) is \( \frac{1}{4\pi} \). When a sine wave represents sound, doubling the frequency results in a pitch one octave higher. So the graph of \( y = \cos x \) represents a sound with pitch one octave higher than the pitch of \( y = 5 \cos \left( \frac{x}{2} \right) \).

**Example 3**

Suppose a tuning fork vibrates with a frequency of 440 cycles per second. If the vibration displaces air molecules by a maximum of 0.3 mm, give a possible equation for the sound wave that is produced.

10. Suppose one tone has a frequency of 440 cycles per second, and another tone has a frequency of 880 cycles per second.
   a. Which has the higher pitch?
   b. How much higher is that pitch?

11. Consider a tuning fork vibrating at 512 cycles per second and displacing air molecules by a maximum of 0.14 mm. Give a possible equation for the sound wave that is produced.

12. **Multiple choice.** A sound wave whose parent graph is \( y = \sin x \) has five times the frequency and is four times as loud as the parent. What is a possible equation for this sound wave?
   (a) \( y = 5 \sin 4x \)  (b) \( y = 4 \sin 5x \)
   (c) \( y = 4 \sin \frac{x}{2} \)  (d) \( y = \frac{4}{5} \sin \frac{x}{3} \)