Hyperbola: set of all points in a plane where the difference between the distances from any point to 2 fixed points (foci) is a constant. Two branches of a hyperbola.

Foci: $F_1$, $F_2$

Vertex: $V_1$, $V_2$

Distance from center to vertex $a = \frac{2a}{V_1} \rightarrow V_2$

Distance from center to focus $c = \frac{2c}{F_1} \rightarrow F_2$

Distance from center to top of rectangle $b = \frac{2b}{C}$

$a^2 + c^2 = \frac{e^2}{b}$

Constructing a hyperbola:

1) Calculate $a$, $b$, $c$ from info.
2) Construct a rectangle, length $2a$ wide and $2b$ tall with center $\pm a$, center $\pm b$.
3) Draw diagonals through rectangle & extend lines. Lines will be the asymptotes of the hyperbola.
4) Starting with each vertex, construct branches of the hyperbola with branches approaching asymptotes.

Hyperbola standard equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Centered at origin, oriented horizontally

$y = 0$ (x-axis)

Major (focal) axis
\[ \frac{y^2}{a^2} - \frac{x^2}{c^2} = 1 \]

Centered @ origin, oriented vertically, 
X=0 (y-axis) is major (focal) axis.

\[ \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{c^2} = 1 \]

Centered @ (h, k), 
oriented horizontally.

\[ \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{c^2} = 1 \]

Centered @ (h, k), 
oriented vertically.

Notice: ellipse: \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

Hyperbola: \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

e: ellipse: larger denominator was \( a^2 \) &
edetermined orientation horiz. or vertically.

hyperbola: first term (before subtraction sign) determines orientation \( \geq \) or \( \leq \).

if \( a^2 \geq c^2 \) then hyperbola branches are flatter.

if \( a^2 \leq c^2 \) then hyperbola branches are sharper.

e > 1

asymptote equations: \[ y = \pm \frac{a}{c} x \]
Hyperbola

\[ \frac{x^2}{4} - \frac{y^2}{9} = 1 \]

\[ a^2 = 4 \quad a = 2 \]
\[ b^2 = 9 \quad b = 3 \]
\[ c^2 = 13 \quad c = \sqrt{13} \]

Oriented horizontally because \( x^2 \) is first

Centered at origin

Asymptotes \( y = \pm \frac{3}{2} x \)

\[ V_1(-2,0) \quad V_2(2,0) \]
\[ F_1(-\sqrt{13},0) \quad F_2(\sqrt{13},0) \]

Find equation of hyperbola with center \( \odot (0,2) \)

Vertex \( \odot (0,6) \)

One asymptote \( y = \frac{2}{3} x + 2 \)

\[ a = 4 \quad y = \frac{a}{c} \quad \frac{2}{3} = \frac{y}{6} \]

If \( a = 4 \)

\[ c^2 = a^2 + c^2 \]
\[ 16 + 36 \]
\[ C^2 = 52 \]
\[ C = \sqrt{52} \]

\[ F_1 \odot (0, 2 + \sqrt{52}) \]
\[ (0, 2 + 2\sqrt{13}) \]

\[ F_2 \odot (0, 2 - 2\sqrt{13}) \]

\[ \frac{(y-2)^2}{16} - \frac{x^2}{36} = 1 \]
\[
\frac{x^2}{1} - \frac{y^2}{4} = 1
\]

Hyperbola oriented along the x-axis

Transverse/major axis

\( \frac{x}{a} + \frac{y}{b} = 1 \)

\( a^2 + b^2 = c^2 \)

\( a = 1 \quad b = 2 \)

\( 1 + 4 = 5 \quad c = \sqrt{5} \)

Asymptotes:

\[ y = \pm \frac{b}{a} \pm 2x \]

\[
X^2 + 9y^2 + 2x + 36y - 44 = 0
\]

\[
X^2 + 2x + 1 - (9y^2 - 36y + \text{constant}) = 0
\]

\[
X^2 + 2x + 1 - 9(y^2 - 4y + 4) = 44
\]

\[
(X + 1)^2 - 9(y - 2)^2 = 36
\]

\[ \frac{(X + 1)^2}{9} - (y - 2)^2 = 1 \]

\[ C = -1, 2 \]

\( a_x = 9 \quad b = 1 \)

\( a = 3 \quad b = 1 \)

\[ c^2 = a^2 + b^2 = 10 \]

\[ c = \sqrt{10} \]

\[ y = \pm \frac{b}{a}x \pm \frac{1}{3}x \]

\[ y - 2 = \frac{1}{3}(x + 1) \]

\[ y - 2 = \frac{1}{3}x + \frac{1}{3} \]

\[ y = \frac{1}{3}x + \frac{7}{3} \]

\[ y = -\frac{1}{3}x + \frac{5}{3} \]