Quick and easy

1. Complete the square:
   \[
   \left(x + 9\right)^2 + \left(y - \frac{7}{2}\right)^2 + 2\left(z^2 - \frac{1}{2}z + \frac{1}{16}\right) = 2z^2 - z + \_
   \]

2. Solve the system:
   \[
   \begin{align*}
   2x - y &= 0 \\
   x^2 + y^2 &= 20
   \end{align*}
   \]

   \[
   \begin{align*}
   y &= 2x \\
   x^2 + (2x)^2 &= 20 \\
   5x^2 &= 20 \\
   x^2 &= 4 \\
   x &= \pm 2 \\
   \end{align*}
   \]

   \[
   (2, 4), (-2, -4)
   \]

Multi-step and not so easy

Standard equation for a circle: 
\[
(x - h)^2 + (y - k)^2 = r^2
\]
where \((h, k)\) is the center of the circle, and \(r\) is the radius.

3. Find an equation of the line tangent to the circle \(x^2 + y^2 + 4x - 46 = 0\) at the point \(P(3, 5)\).
   
   Suggestion: find a standard equation of the circle, and sketch the circle and point on Cartesian coordinate axes.

   \[
   \begin{align*}
   (x^2 + 4x + \_
   \]

   \[
   \begin{align*}
   (x^2 + 4x + 4) + y^2 &= 46 + 4 \\
   (x + 2)^2 + y^2 &= 50
   \end{align*}
   \]

   Center of circle \(C(-2, 0)\)

   \[
   \frac{5 - 0}{3 - (-2)} = 1
   \]

   \[
   m_{\text{radius from center to } (3, 5)} = \frac{5 - 0}{3 - (-2)} = 1
   \]

   \[
   \text{m of tangent line} = -1
   \]

   \[
   \therefore \text{equation of tangent line}
   \]

   \[
   y - 5 = -1 (x - 3)
   \]

   \[
   \begin{align*}
   y &= -x + 3 \\
   y &= -x + 8
   \end{align*}
   \]

   \[
   x + y = 8
   \]

   Standard form
4. Find an equation of the circle that passes through \((-7, 4)\) and has y-intercepts \((0, 11)\) and \((0, -13)\).

Standard equation for a circle: \((x - h)^2 + (y - k)^2 = r^2\) where \((h, k)\) is the center of the circle, and \(r\) is the radius.

Suggestion: Sketch the 3 points of the circle on Cartesian coordinate axes.

- bisectors must \(\cap\) at center of the circle
- center of circle must lie on the midline between 2 points
- midline between \((0, 11)\) & \((0, -13)\) is \(y = -1\)
- center of circle must lie on \(y = -1\)
- \(\perp\) bisector of \((-7, 4)\) and \((0, 11)\) must intersect \(y = -1\) @ center of circle
- \(\perp\) bisector of \((-7, 4)\) & \((0, 11)\) \(\cap\) midpoint @ \(\left(-\frac{7}{2}, \frac{15}{2}\right)\)

Midline from \((-7, 4)\) to \((0, 11)\) is \(\frac{11-4}{0-(-7)} = 1\) \(\Rightarrow\) \(\perp\) line \(y - \frac{15}{2} = -1 \cdot (x + \frac{7}{2})\)

Equation of \(\perp\) bisector: \(y - \frac{15}{2} = -1 \cdot (x + \frac{7}{2})\)

\(y = -x + 4\)

\(-1 = -x + 4\)

\(x = 5\)

Center of circle @ \((5, -1)\)

Length of radius \(\sqrt{(5-0)^2 + (-1-11)^2} = \sqrt{169} = 13\)

Equation of circle: \((x - 5)^2 + (y + 1)^2 = 169\)

Alternatively: solve a system of equations:
\[
\begin{align*}
(x - h)^2 + (y - k)^2 &= r^2 \\
(-h)^2 + (y - k)^2 &= r^2 \\
(-h)^2 + (-13 - k)^2 &= r^2
\end{align*}
\]

\[
\begin{align*}
h^2 + 22k + k^2 &= r^2 \\
h^2 + 169 + 26k + k^2 &= r^2
\end{align*}
\]

\[48k = -48\]

\[k = -1\]