Section 6.2 Equations of Circles

Objective: to find equations of circles and find the coordinates of any points of where a circle and line intersect.

Given a point \((x,y)\) and the center of the circle \((h,k)\) and radius \(r\). The equation of a circle is:

\[(x-h)^2 + (y-k)^2 = r^2\]  \(\text{and} \quad \sqrt{(x-h)^2 + (y-k)^2} = r\)

If the center of a circle is the origin then \((x-0)^2 + (y-0)^2 = r^2\)

simplifies to \(x^2 + y^2 = r^2\)

\((h,k)\) represents the horizontal \((h)\) and vertical shift \((k)\) of the circle from the origin.

- **Example One**  
  Given the equation of a circle \((x - 2)^2 + (y - 3)^2 = 25\)  
  the center of the circle is \((2,3)\) and has a radius of \(\sqrt{25} = 5\).

- **Example Two**  
  Given the equation of a circle \((x - 2)^2 + (y + 5)^2 = 22\)  
  the center of the circle is \((2, -5)\) and has a radius of \(\sqrt{22}\).

- **Example Three**  
  Given the equation of a circle \(x^2 + y^2 +10x -4y -20 = 0\).  
  To find the center and radius of the circle you must rewrite by completing the square.  
  Rewrite the equation as \(x^2+10x + y^2 -4y -20=0\)  
  Complete the square of both the \(x\) terms and the \(y\) terms.  
  Remember \((b/2)^2\)

  \[
  \frac{(x^2+10x + \frac{25}{2})}{49} + \frac{(y^2 - 4y + 4)}{4} = 20 + \frac{25}{4} + 4
  \]

  The center of the circle is \((-5,2)\) and the radius is \(\sqrt{49} = 7\).

- **To graph a circle:** solve for \(y\) in terms of \(x\).  
  Given the equation : \((x + 5)^2 + (y - 2)^2 = 49\)  
  \((y - 2)^2 = 49 - (x+5)^2\)  
  Take the square root of both sides \(\sqrt{(y-2)^2} = \sqrt{49 - (x+5)^2}\)  
  \(y - 2 = \pm \sqrt{49 - (x+5)^2}\)  
  Add 2 to both sides \(y = 2 \pm \sqrt{49 - (x+5)^2}\)  
  Graph each solution. Adjust your window to reduce distortion of the circle.

- **Find the intersection of a circle and a line algebraically, if any.**  
  Remember there are three possibilities!!! (one point, two points or no points)  
  Given: a line \(y = 2x -2\) and a circle \(x^2 + y^2 = 25\)  
  Use the equation of the line and substitute the expression for \(y\) in the equation of the circle \(x^2 + (2x -2)^2 = 25\) and solve.

  \[
  x^2 + 4x^2 - 8x + 4 = 25
  \]

  \(5x^2 - 8x - 21 = 0\)

  Factor and solve \((5x +7)(x-3) = 0\)

  \(x = -7/5 \quad \text{and} \quad x = 3\)

  Substitute each value of \(x\) into the equation of the line to find the coordinates of the points of intersection. When \(x = -7/5, \quad y = 2(-7/5) -2\). The pt of intersection is \((-7/5, -24/5)\)

  And when \(x = 3, \quad y = 2(3) -2\). The pt. of intersection is \((3, 4)\)