Adding, Subtracting, Multiplying, and Dividing Polynomials

I. To add or subtract terms with variables (monomials), the terms must be “like”;
   i.e. the terms must have the same variable base to the same degree (raised to the same exponent)
   i.e. the power (base plus exponent) must be the same.

\[ x^2 + x^3 \] cannot be simplified \[ x + x^2 \neq x^3 \], just as \[ 3 + 3^2 \neq 3^3 \]

\[ ab + b \] cannot be simplified \[ a^2 + a^2b + ab^2 + a^2b^2 \] cannot be simplified

\[ a^2 + 3a^2b + 4a^2 + 5a^2b = 5a^2 + 8a^2b \] (\(a^2\)'s are like terms; \(a^2b\)'s are like terms)

\[ (5x^2 + 2x + 1) - (x^2 + 3x - 2) = 4x^2 - x + 3 \] (\(x^2\)'s are like terms; \(x\)'s are like terms; numbers are...)

II. Zero Exponent: Any power with a zero exponent = 1

E.g. \(4^0 = 1\) \(\frac{1}{2}^0 = 1\) \((abc)^0 = 1\) \((0^0\) is undefined) \(-7^0 \neq 1\), \(-7^0 = -1\)

III. Negative Exponent: Any power with a negative exponent can be simplified by canceling the negative sign and placing the remainder of the power on the opposite side of the fraction bar.
The negative sign in the exponent does not make the number negative.

E.g. \(2^{-1} = \frac{1}{2} = \frac{1}{2}\) \(b^{-3} = \frac{1}{b^3}\) \(\frac{1}{2^{-2}} = 2^2 = 4\) \((0.1)^{-3} = (\frac{1}{10})^{-3} = 1000\)

IV. Multiplying Powers: To multiply powers with the same base, keep the base and add the exponents.

\[ x^2 \cdot x^3 = x^{2+3} = x^5 \]
\[ x^{-2} \cdot x^2 = x^{-2+2} = x^0 = 1 \]

To multiply powers with different bases but the same exponent, multiply the bases and keep the exponent.

\[ 2^2 \cdot 3^2 = 6^2 \]
\[ x^2 \cdot y^2 = (xy)^2 \]

To raise a power to a power, keep the base and multiply the exponents.

\[ (x^2)^3 = x^{2\cdot3} = x^6 \]
\[ (x^{-2})^3 = x^{-2\cdot3} = x^{-6} = \frac{1}{x^6} \]

To raise a product to a power, multiply the power by the exponent of each base. (Power of a product

\[-\text{Product of the powers}\]

\[ (ab)^2 = a^2b^2 \]
\[ 5(ab)^3 = 5a^3b^6 \]
V. Dividing Powers: To divide powers with the same base, keep the base and subtract the exponents.

\[
\frac{a^3}{a} = a^{3-1} = a^2 \quad \frac{b^2}{b^5} = b^{2-5} = b^{-3} = \frac{1}{b^3} \quad \frac{c^0}{c^{-3}} = c^{0-(-3)} = c^3
\]

In the alternative, with positive exponents, you can compare the exponents in the numerator and denominator, and cancel the common factors.

\[
\frac{a^3}{a} = \frac{a \cdot a \cdot a}{a} = a^2
\]

Multiplying polynomials: multiply the coefficients separately, then multiply each variable separately.

VI. Multiplying a monomial by a polynomial: multiply the monomial by each term of the polynomial in order.

E.g. \(3ab(2a^2 - 4b + c) = 6a^3b - 12ab^2 + 3abc\)

VII. Multiplying a binomial by a polynomial: multiply the first term of the binomial by the first term of the polynomial, then multiply the first term of the binomial by the second term of the polynomial, and continue until the first term of the binomial has been multiplied with each term of the polynomial; then multiply the second term of the binomial with each term of the polynomial; then combine like terms to simplify the answer.

E.g. \((2a - 3b)(a + 2b - 3c) = 2a^2 + 4ab - 6ac - 3ab - 6b^2 + 9bc = 2a^2 + ab - 6ac - 6b^2 + 9bc\) (ab's are like terms)

A mnemonic device to remember the order for multiplying two binomials is FOIL (first, outer, inner, last)

E.g. \((x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9\)

- No shortcuts: \((x - 3)^2 \neq x^2 - 9, \& \neq x^2 + 9\) \((x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9\) (don’t lose the middle term)

Also \((a + b)^3 \neq a^3 + b^3\)