Practice Problems for Transformations of Functions

1. If $n = f(A)$ gives the number of gallons of paint needed to cover an area of $A$ sq.ft., explain the meaning (in the context of painting) of:

(a) $f(A+10)$ number of gallons needed to cover $(A+10)$ sq.ft.
(b) $f(A)+10$ number of gallons needed to cover $A$ sq.ft. plus ten more gallons of paint.

2. The graph of $f(x)$ contains the point $(3, -4)$. What point must be on the graph of:

(a) $f(x)+5$  
(b) $f(x)+5$  
(c) $f(x-3)-2$  
(d) $\frac{1}{2}f(x)$  
(e) $f(\frac{1}{2}x)$

(f) $-3f(x)$  
(g) $f(-3x)$  
(h) $-f(\frac{1}{2}x-3)+1 = -f(\frac{1}{2}(x-9))+1$

3. Table 6.3 contains values of $g(t)$. Find formulas in terms of $g(t)$ for each of the functions in tables a – e.

<table>
<thead>
<tr>
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<tr>
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<td>1.0</td>
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(a) $t$  
(b) $t$  
(c) $t$  
(d) $t$  
(e) $t$

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<tr>
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<td>1.7</td>
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</table>
4. \( H(t) = 68 + 93 \cdot (0.91)^t \) gives the temperature in degrees Fahrenheit (°F) of a cup of coffee t minutes after it is brought into a classroom.

(a) What was the temperature of the coffee when it was first brought into the classroom? \( 61^\circ F \).

(b) What was the temperature of the classroom? \( 68^\circ F \).

(c) How fast does the temperature difference between the coffee and the room change? \( 9^\circ \) per minute.

(d) Find a formula for \( H(t + 15) = 68 + 93 \cdot (0.91)^{t+15} \).

(e) Find a formula for \( H(t) + 15 = 68 + 93 \cdot (0.91)^t + 15 = 83 + 93 \cdot (0.91)^t \).

(f) Describe in practical terms a situation modeled by \( H(t + 15) \) and a situation modeled by \( H(t) + 15 \).

\( H(t + 15) \) is \( H(t) \) shifted 15 units to the left. \( H(t + 15) \) could represent the temperature of a cup of coffee brought into the classroom 15 minutes earlier.

\( H(t) + 15 \) is \( H(t) \) shifted 15 units up. \( H(t) + 15 \) could represent the temperature of a cup of coffee brought into a warmer (68°F warmer).

(g) Which function, \( H(t + 15) \) or \( H(t) + 15 \), approaches the same final temperature as \( H(t) \)?

As \( t \to \infty \) (gets larger), \( H(t) \) and \( H(t + 15) \) approach 68°F (room temperature).

\( H(t) + 15 \) approaches 83°F (its room temp.)

5. A hot pottery bowl is removed from a kiln and set on the floor to cool. The difference \( D(t) \) between the pot's temperature, initially 350°F, and the room temperature, 70°F, decays exponentially over time at a rate of 3% per minute. The pot's temperature \( P(t) \) is a transformation of \( D(t) \).

\( D(t) = 280 \cdot (0.97)^t \)

(a) Find a formula for \( P(t) \). \( P(t) = 280 \cdot (0.97)^t + 70 \)

(b) Sketch graphs of \( D(t) \) and \( P(t) \) on the same set of axes.

Label the equations of the asymptotes on the graph.

Alternative method for \( \ln(5e^x) = \ln(e^{x+h}) \)

\( \ln(5e^x) = \ln(e^{x-h}) \)

\( \ln(5) + \ln(e^x) = x - h \)

\( \ln 5 + x = x - h \)

6. \( f(x) = e^x \) and \( g(x) = 5e^x \). If \( g(x) = f(x - h) \), find \( h \).

\( 5e^x = e^{x-h} \)

\( 5e^x = e^x \cdot e^{-h} \)

\( h = \ln \left( \frac{1}{e^h} \right) \)

\( h = \ln \left( \frac{1}{e} \right) \) or 0.2

\( h = \ln 0.2 \)
7. A function \( Q(t) \) has domain \( t \geq 0 \), \([0, \infty)\) in interval notation, and range \(-4 \leq Q(t) \leq 7\), or \([-4, 7]\).

State the domain and range in interval notation for:

(a) \( y = Q(-t) \)
   \[ \text{D: } (-\infty, 0] \quad \text{R: } [-4, 7] \]

(b) \( y = -Q(t) \)
   \[ \text{D: } [0, \infty) \quad \text{R: } [-7, 4] \]

(c) \( y = -Q(-t) \)
   \[ \text{D: } (-\infty, 0] \quad \text{R: } [-7, 4] \]

(d) \( y = -Q(t - 4) \)
   \[ \text{D: } [4, \infty) \quad \text{R: } [-7, 4] \]

8. State a formula for each of the transformations of \( m(n) = n^2 - 4n + 5 \).

(a) \( y = m(-n) \)
   \[ n^2 + 4n + 5 \]

(b) \( y = -m(n) \)
   \[ -n^2 + 4n - 5 \]

(c) \( y = -m(-n) \)
   \[ -n^2 - 4n - 5 \]

(d) \( y = m(-n) + 3 \)
   \[ n^2 + 4n + 8 \]

9. Using Figure 6.24, calculate:

(a) \( f(-x) \) for \( x = -4 \)
   \[ -10 \]

(b) \( -f(x) \) for \( x = -6 \)
   \[ -25 \]

(c) \( -f(-x) \) for \( x = -4 \)
   \[ 10 \]

(d) \( -f(x + 2) \) for \( x = 0 \)
   \[ 4 \]

(e) \( f(-x) + 4 \) for \( x = -6 \)
   \[ -26 \]
10. The function \( n = f(A) \) represents the number of gallons of paint needed to cover an area of \( A \) sq. ft.

Generate a transformation of \( n = f(A) \) to represent the following scenarios:

(a) I calculated how many gallons I needed to cover \( A \) and bought 2 more gallons than I needed. \( n = f(A) + 2 \)
(b) I bought enough paint to apply two coats of paint on the area \( A \). \( n = 2f(A) \)
(c) I bought enough paint to cover the area \( A \) plus two square feet more. \( n = f(A + 2) \)

11. The U.S. population in millions is \( P(t) \) today, \( t \) in years.

Match each of the following statements (I – IV) with one of the following formulas (a – h):

I. The population ten years before today. \( d \)
II. Today’s population plus 10 million immigrants. \( c \)
III. Ten percent of today’s population. \( f \)
IV. The population after 100,000 people have emigrated. \( h \)

(a) \( P(t) - 10 \) (b) \( P(t - 10) \) (c) \( P(t) + 10 \) (d) \( P(t + 10) \) (e) \( P(t) / 0.1 \) (f) \( 0.1P(t) \) (g) \( P(t) + 0.1 \) (h) \( P(t) - 0.1 \)

12. Table 6.17 contains values of \( f(x) \).

Each subsequent function, \( g(x) = m(x) \), can be obtained by applying a single transformation to \( f(x) \).

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<tbody>
<tr>
<td>( m(x) )</td>
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<td>0</td>
<td>-12</td>
<td>-18</td>
<td>-6</td>
<td>-4</td>
<td>6</td>
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</table>
13. Write a formula for each of the transformations of \( Q(t) = 4e^{6t} - 3 \):
   
   \( a. \ Q\left(\frac{t}{4}\right) = 4e^{\frac{2t}{4}} - 3 \quad b. \ Q(t) \cdot \frac{4}{3} = e^{\frac{6t}{4}} - 1 \quad c. \ Q(2t) + 11 = 4e^{12t} + 8 \quad d. \ Q(t - 3) = 28e^{0.6(t-3)-3} \)

14. The function \( f(x) \) represents the daily cost to me of my Uber driver driving a total of \( x \) miles (to and from work). Write a transformation of \( f(x) \) to represent the Uber cost to me for each scenario:
   
   (a) I received a raise yesterday, so I gave my Uber driver a $5.00 tip today. \( \frac{f(x) + 5}{\frac{5}{f(x)}} \)
   
   (b) I haven't paid my driver all week, so I owe him/her for five days of driving. \( f(x + 5) \)
   
   (c) There was an accident on the way to work today so my driver had to take a detour which added five extra miles to the trip to work. \( f(5x) \)
   
15. \( A = f(r) \) represents the area of a circle of radius \( r \).
   
   (a) Write a formula for \( f(r) \). \( \pi r^2 \)
   
   (b) Which expression represents the area of a circle whose radius is increased by 10%?
   
   (i) \( 0.10 f(r) \) \hspace{1cm} (ii) \( f(r + 0.10) \) \hspace{1cm} (iii) \( f(0.10r) \) \hspace{1cm} (iv) \( f(1.10r) \) \hspace{1cm} (v) \( f(r) + 10 \)
   
   (c) If the radius is increased by 10%, by what percent is the area increased? \( 210\% \)

16. The function \( f(x) \) has domain \( -6 \leq x \leq 2 \), \([-6, 2]\) interval notation, \( \{x | -6 \leq x \leq 2\} \) in set notation. The average rate of change of \( f(x) \) over that domain is \( \frac{2.4}{2} = 3 \).

   For each of the following transformations of \( f(x) \), state the new domain and average rate of change.
   
   (a) \( f(2x) \) \hspace{1cm} D: \([-3, 1]\) \hspace{1cm} Avg. Rate of change: \( \frac{+2.4}{4} = 0.6 \)
   
   (b) \( f\left(\frac{1}{2}x\right) \) \hspace{1cm} D: \([-2, 8]\) \hspace{1cm} Avg. Rate of change: \( \frac{+2.4}{3.2} = \frac{3}{4} \)
   
   (c) \( f(x+2) \) \hspace{1cm} D: \([-8, 0]\) \hspace{1cm} Avg. Rate of change: \( 3 \)
   
   (d) \( f(-x) \) \hspace{1cm} D: \([-2, 6]\) \hspace{1cm} Avg. Rate of change: \( -3 \)
17. Fill in as many values as possible:

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<tr>
<td>2f(x) + 3</td>
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<td>-5</td>
<td>7</td>
<td>3</td>
<td>13</td>
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<tr>
<td>f(x - 1) + 1</td>
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<tr>
<td>f(x + 2) - 1</td>
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<td>-1</td>
<td>4</td>
<td>X</td>
<td>X</td>
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<tr>
<td>3f(2x + 2) - 1</td>
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<td>X</td>
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18. The graph of \( f(x) \) below has x-intercepts \((-1, 0)\) and \((3, 0)\), y-intercept \((0, -2)\), and a horizontal asymptote at \( y = 2 \).

For the transformations, find the: x-intercepts, y-intercept, and horizontal asymptote

(a) \(3f(x)\)  
\((-1, 0), (3, 0)\)  
\((0, -6)\)  
\(y = 6\)

(b) \(f(x-1)\)  
\((0, 0), (4, 0)\)  
\((0, 0)\)  
\(y = 2\)

(c) \(f(x) - 1\)  
\(X\) \(X\)  
\((0, -3)\)  
\(y = 1\)

(d) \(-2f(x)\)  
\((-1, 0), (3, 0)\)  
\((0, 4)\)  
\(y = -4\)

(e) \(\frac{1}{2}f(x+2) - 1\)  
\(\approx (0, -2)\)  
\(y = 0\)

(f) \(-f(-x)\)  
\((-3, 0), (1, 0)\)  
\((0, 2)\)  
\(y = -2\)
19. The points \((-12, 20), (0, 6), \text{ and } (36, -2)\) lie on the graph of \(f(x)\). Find the corresponding points on the graph of \(g(x) = 10 - 2f(-3x)\).

\[
(4, -30) \quad (0, -2) \quad (-12, 14)
\]

20. \(f(x) = e^x\) and \(g(x) = e^{x-2}\). If \(g(x) = kf(x)\), find \(k\).

\[
k e^x = \frac{e^x}{e^2} \quad \Rightarrow \quad k = \frac{1}{e^2}
\]

21. If \(h(x) = \log(ax)\), what is \(k\) if \(h(x) = \log(x) + k\)?

\[
\log (ax) = \log a + \log x \quad \Rightarrow \quad k = \log a
\]

22. In the table below, fill in all the boxes for which you have sufficient information.

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<tr>
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<td>-6</td>
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<td>(X)</td>
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23. Match each of the functions (a – f) with one of the graphs (i – vi) below:

(a) \( y = e^x \)  
(b) \( y = e^{5x} \)  
(c) \( y = 5e^x \)  
(d) \( y = e^{x+5} \)  
(e) \( y = e^{-x} \)  
(f) \( y = e^x + 5 \)

With thanks to Professor Eric Connally, *Functions Modeling Change.*