### Solving Polynomial Inequalities - 2 Methods

#### A. Sign analysis

1. **Factor the polynomial to find the zeros & vertical asymptotes**
2. **Plot the zeros & vertical asymptotes on a number line.**
3. **If \( n < 0 \), use hollow circle for zeros (not part of solution).**
   **(always use "" for vertical asymptotes)**
4. **If \( n \geq 0 \), use filled in circle for zeros (part of the solution).**
5. **For each interval: from \(-\infty\) to the least zero, between each pair of zeros, & from the greatest zero to \(\infty\), pick a \# & substitute that \# in each factor to determine if each factor is +/-; then assign +/- to the product of the factors & enter +/- in the interval on the number line.**
6. **Determine which intervals satisfy the inequality.**

#### B. same as above

1. **same "**
2. **same "**
3. **same "**
4. **Sketch a graph of the polynomial using end behavior**
5. **same as above**

#### Example 1

\( (2x + 5)(x - 1) < 0 \)

- **\( x = -\frac{5}{2}, 1 \)**
- **\( -3 \) - - - = +
- **\( 0 \) + - = -
- **\( 2 \) + + = +

- **\( -\infty \) \( \rightarrow \) \( \frac{-5}{2} \) \( \rightarrow \) \( 0 \) \( \rightarrow \) \( 1 \) \( \rightarrow \) \( \infty \)**

#### Example 2

- **\( + \)** quad. for \( \rightarrow \end{behavior}

- **Solution: \( -\infty \) \( \rightarrow \) \( \frac{5}{2} \) \( < x < 1 \) \( \rightarrow \) \( \infty \)**
For rational expressions with polynomial numerators and denominators (creating vertical asymptotes), you probably do not know the behavior of the graph, so we have to resort to sign analyses to solve the polynomial inequality.

When solving a rational expression inequality with a polynomial in the denominator (or numerator), the graph may not be obvious, so we resort to sign analyses.

Assuming no graphing calc.

E.g., p. 103 #28 \( \frac{3n^2 - 12}{3n^2 - 12} \leq 0 \)

What is the graph?

1. \( \text{numerator} = 0 \) when \( n = 4 \) \( \Rightarrow \) zero @ \((4,0)\)
2. \( \text{denominator} = 0 \) when \( 3n^2 = 12 \) \( \Rightarrow \) double root @ \(4,0\) (squared factor in num.)
   \( n^2 = 4 \)
   \( n = \pm 2 \) \( \Rightarrow \) vertical asymptote @ \( x = \pm 2 \)
3. Highest powers of the variable \( n^2 \) in both num. & denom.
   \( \Rightarrow \) horizontal asymptote \( y = \frac{(3n^2)}{3n^2} = \frac{9n^2}{3n^2} = 3 \)

Since it is a rational (not an irrational) expression, the horizontal asymptote is the same on either side \( +/ - \infty \).

You still don’t know what the graph is without testing values.

\( +/- \)

Sign analysis: \( n = 4 \) is the only zero & it is part of the solution \( \leq 0 \).

It is a double zero, so the same sign to left & right of \( 4 \) must check to either side of vertical asymptotes: signs could be the same or different \( \frac{3}{1} \)

Squared factors are always positive,
so only need to check values with denominator

\[
\begin{array}{c|cccc}
 n & -3 & 0 & 3 & 5 \\
 \hline
 3n^2 - 12 & - & + & + & + \\
 \end{array}
\]

NB: Vertical asymptote can never be part of the solution.

Solution: \( -2 < x < 2 \) or \( x = 4 \)