Solutions to

Pre-calculus Worksheet: Solving for a variable in a radical, extraneous solutions §1.6

To solve for a variable in a radical: 1. Isolate the radical  2. Square both sides of the equation.  3. Solve for the variable  4. Check for extraneous solutions

Whenever you raise a term to an even power, e.g. squaring, fourth power (^4), three-halves power (^\frac{3}{2}), you may introduce an extraneous solution. Extraneous solutions may also be generated when you cross-multiply to solve a proportion. Look for extraneous solutions that don’t work in the original equation or that make denominators zero.

Solve: \( \sqrt{3x+6} = 2x+1 \)

\[
3x + 6 = 4x^2 + 4x + 1
\]
\[
0 = 4x^2 + x - 5 \quad \frac{-20}{-4 + 5}
\]
\[
0 = (4x + 5)(x - 1)
\]
\[
X = \frac{-5}{4}, 1
\]

Check

\( \text{@} \ X = \frac{-5}{4} \)

\[
\sqrt{3\left(-\frac{5}{4}\right) + 6} = 2\left(-\frac{5}{4}\right) + 1
\]

\[
\frac{\sqrt{-15} + \frac{24}{4}}{4} = -\frac{10}{4} + \frac{4}{4}
\]

\[
\text{reject} - \frac{5}{4} \text{ as extraneous}
\]

Solve: \( \frac{x+4}{6x-3} = \frac{8-x}{5x-1} \)

\[
(5x-1)(x+4) = (6x-3)(8-x)
\]
\[
5x^2 + 20x - x - 4 = 48x - 6x^2 - 24 + 3x
\]
\[
11x^2 - 32x + 20 = 0
\]
\[
(11x - 10)(x - 2) = 0
\]
\[
X = \frac{10}{11}, 2
\]

Check

\( \text{@} \ X = \frac{10}{11} \)

\[
\frac{10}{11} + \frac{4}{11} \quad \frac{8}{11} - \frac{10}{11}
\]
\[
\frac{60}{11} - \frac{33}{11} = \frac{50}{11} - \frac{11}{11}
\]
\[
\frac{54}{27} = \frac{78}{39}
\]
\[
2 = 2
\]
Solve: $x - \sqrt{15 - 2x} = 0$

$X = \sqrt{15 - 2x}$

$X^2 = 15 - 2x$

$X + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5, 3$

**Checks**

@ $x = -5$

$-5 = \sqrt{15 - 2(-5)}$

$-5 = \sqrt{15 + 10}$

$-5 = \sqrt{25}$

$-5 \neq 5$

$x = -5$ is extraneous.

@ $x = 3$

$3 = \sqrt{15 - 2(3)}$

$3 = \sqrt{9}$

$3 = 3 \checkmark$

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Solve: $\frac{x+1}{x-3} = \frac{3}{x} + \frac{12}{x^2 - 3x}$

$X(x+1) = \frac{3(x-3)}{(x)(x-3)} + \frac{12}{x(x-3)}$

$X^2 + x = 3x - 9 + 12$

$X^2 - 2x + 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

**Check**

@ $x = -1$

$-1 + 1 = \frac{3}{-1} + \frac{12}{(-1)^2 - 3(-1)}$

$-1 - 3 = -1 + \frac{12}{1+3}$

$0 = -3 + 3$

$0 = 0 \checkmark$

reject 3 as extraneous because it makes the denominators in the original equation 0.