Your summer assignment is:

1. Read and study the six pages of notes and examples entitled “Pre-calculus Prerequisites”.
   You will be tested on the algebra in those six pages within one week of the start of the school year.

2. Read and study the two pages of class notes entitled “Linear Functions” covering § 1.1 – 1.4 of the
   Brown textbook titled Advanced Mathematics.

3. Complete the 3-page worksheet entitled “Linear Equations Worksheet 1”. Answers on separate paper.
   Your answers to Linear Equations Worksheet 1 will be collected on the first day of school.

4. A 2-page sample of Linear Equations Worksheet 1, completed for the points (2, -3) and (-1, 1), is
   attached.

5. Read and study the notes on “Solving Quadratic Equations Algebraically: Completing the Square”,
   with three examples of completing the square.

6. Complete the “Solving Quadratic Equations Worksheet”.
   Your answers to the Solving Quadratic Equations Worksheet will be collected on the first day of school.

7. Read and study the notes: “Pre-calculus - Complex Numbers Class Notes”.

8. Complete the “Complex Numbers Worksheet”.
   Your answers to the Complex Numbers Worksheet will be collected on the first day of school.

9. Read and study the notes: Pre-calculus: Proper Notation.

10. Complete the “Graphing Quadratic Functions Worksheet”
    Your answers to the Graphing Quadratic Functions Worksheet will be collected on the first day of school.
**Pre-calculus Pre-requisites:** Some algebra operations/manipulations you should (and must) know (along with common misconceptions about algebra operations)

You can cross-multiply only across an equal sign, never across a $\,+$, $\,-$, $\,\times$, or $\,\div$ sign.  
\[ \frac{5}{x} = \frac{3}{10} \quad \text{E.g.} \quad 50 = 3x \]

You can cross-reduce (cross-cancel) only across a multiplication ($\times$) sign, never across a $\,+$, $\,-$, $\,\div$, or $\,=$ sign.  
\[ \frac{5}{x} = \frac{3}{10} \quad \text{E.g. cannot cancel} \quad \frac{5}{x} = \frac{3}{10} \]

You can reduce a fraction, but you can only reduce a polynomial if the same polynomial appears as a factor in both the numerator and denominator (i.e. no partial canceling of polynomials).

\[ \frac{2x-5}{2} \neq \frac{2x-5}{2} \neq x - 5 \quad \text{E.g.} \quad \frac{x^2-3}{x-3} \neq \frac{x^2-3}{x-3} \neq x \quad \text{E.g.} \quad \frac{2\pm 4i}{2} \neq 1 \pm 4i \neq 2 \pm 2i \]

\[ \frac{2 \pm 4i}{2} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i \]

Dividing by an expression (whether a number, variable, or polynomial) is the same as multiplying by its reciprocal.

\[ \frac{12}{2} \neq 6 \quad \frac{12}{2} = 12 \times \frac{2}{1} = 24 \quad \text{E.g.} \quad \frac{x-1}{2} - \frac{x-1}{2} \quad \frac{1}{2} \quad 2 = (\frac{-1(x-1)}{(x-1)} \quad \frac{1}{2} = \frac{(-1)(x-1)}{(x-1)} \quad \frac{1}{2} = \frac{-1}{2} \]

You can split a rational expression with a single denominator into partial fractions.

\[ \frac{2 + 4}{6} = \frac{2}{6} + \frac{4}{6} = 1 \quad \text{E.g.} \quad \frac{6x + 9}{3} = \frac{6x}{3} + \frac{9}{3} = 2x + 3 \]

You can never split a rational expression with a binomial denominator into partial fractions.

\[ \frac{6}{2 + 4} \neq \frac{6}{2} + \frac{6}{4} \quad \text{E.g.} \quad \frac{12}{3x + 6} \neq \frac{12}{3x} + \frac{12}{6} \neq \frac{4}{x} + 2 \]

Distributive property works for multiplying a binomial (sum or difference of 2 terms). The distributive property does not apply to multiplying a monomial (a product of multiple terms).

\[ 2(x + y) = 2x + 2y \quad \text{E.g.} \quad 2(xy) \neq 2x2y \]

Remember to distribute a negative sign.  
\[ 3 - 2(a - b) \neq 3 - 2a - 2b \]
A line is horizontal if its slope is zero. A rational expression is equal to zero if its numerator is zero (and its denominator is not zero).

Zero slope ≠ no slope. \( \frac{0}{3} = 0 \)

To find when a rational expression = 0, set the numerator = 0 and solve.

A line is vertical if its slope is undefined (no slope). A rational expression is undefined if its denominator is zero.

\( \frac{3}{0} \neq 0 \), it is undefined

To find when a rational expression is undefined, set the denominator = 0 and solve.

**Order of operations:**
1. Parentheses or other grouping symbols
2. Exponents
3. Multiplication or division from left to right
4. Addition or subtraction from left to right

E.g. \( \frac{-1}{2x} \neq 2x^{-2} \)  
E.g. \( 2x^{-1} \neq \frac{1}{2x} \cdot 2x^{-1} = \frac{2}{x} \)  
E.g. \( -4^2 \neq (-4)^2 \)

On the calculator, place fractions, and denominators or exponents with more than one number or variable, in parentheses.

E.g. \( \frac{5}{4\pi} \) must be written as \( 5/(4\pi) \) on the calculator; \( 5/4\pi \) means \( (5/4)\pi \) on the calculator.

A rational expression is a fraction with numbers and/or variables. The rules for fractions apply.

**I. To add or subtract fractions:**

\[ \frac{1}{a} + \frac{1}{b} \neq \frac{2}{a+b} \neq \frac{2}{ab} = \frac{a+b}{ab} \]

(a) if the denominators are the same, add/subtract numerators, and place the sum/difference over the common denominator;

(b) if the denominators are different, convert both fractions over the lowest common denominator, the LCD, and then add/subtract numerators, and place the sum/difference over the LCD.

**II. To multiply fractions:** (a) cross-cancel to reduce fractions, then (b) multiply numerators separately, and multiply denominators separately.

To multiply a fraction by an integer, multiply only the numerator by the integer.

E.g. \( 2 \cdot \left( \frac{3x}{y} \right) \neq \frac{6x}{2y} = \frac{6x}{y} \)

**III. To divide fractions:**

(a) if the fractions are written horizontally, flip the second fraction and multiply the two fractions

(b) if the fraction is written vertically, flip the fraction in the denominator

E.g. \( \frac{3}{4} \div \frac{5}{6} = \frac{3 \cdot 6}{4 \cdot 5} = \frac{9}{10} \)

and multiply that reciprocal fraction by the fraction in the numerator

E.g. \( \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc} \)

Remember that you can cross-cancel only across a multiplication sign, never across a division sign. You can reduce a fraction at any time.
Adding, Subtracting, Multiplying, and Dividing Polynomials

I. To add or subtract terms with variables (monomials), the terms must be “like”;
   i.e. the terms must have the same variable base to the same degree (raised to the same exponent)
   i.e. the power (base plus exponent) must be the same.

\[ x^2 + x^3 \text{ cannot be simplified} \quad x + x^2 \neq x^3, \text{ just as } 3 + 3^2 \neq 3^3 \]

\[ ab + b \text{ cannot be simplified} \quad a^2 + a^2b + ab^2 + a^2b^2 \text{ cannot be simplified} \]

\[ a^2 + 3a^2b + 4a^2 + 5a^2b = 5a^2 + 8a^2b \quad (a^2 \text{'s are like terms}; a^2b \text{'s are like terms}) \]

\[ (5x^2 + 2x + 1) - (x^2 + 3x - 2) = 4x^2 - x + 3 \quad (x^2 \text{'s are like terms}; x \text{'s are like terms}; \text{ numbers are...}) \]

II. Zero Exponent: Any power with a zero exponent = 1

E.g. \[ 4^0 = 1 \quad x^0 = 1 \quad \left(\frac{1}{2}\right)^0 = 1 \quad (abc)^0 = 1 \quad (0^0 \text{ is undefined}) \quad -7^0 \neq 1, \quad -7^0 = -1 \]

III. Negative Exponent: Any power with a negative exponent can be simplified by canceling the negative sign and placing the remainder of the power on the opposite side of the fraction bar.
   The negative sign in the exponent does not make the number negative.

E.g. \[ 2^{-1} = \frac{1}{2} = \frac{1}{2} \quad b^{-3} = \frac{1}{b^3} \quad \frac{1}{2}^{2} = 2^2 = 4 \quad (0.1)^{-3} = \left(\frac{1}{10}\right)^{-3} = 1000 \]

IV. Multiplying Powers: To multiply powers with the same base, keep the base and add the exponents.

\[ x^2 \times x^3 = x^{2+3} = x^5 \quad x^{-2} \times x^2 = x^{-2+2} = x^0 = 1 \]

To multiply powers with different bases but the same exponent, multiply the bases and keep the exponent.

\[ 2^2 \times 3^2 = 6^2 \quad x^2 \times y^2 = (xy)^2 \]

To raise a power to a power, keep the base and multiply the exponents.

\[ (x^2)^3 = x^{2 \times 3} = x^6 \quad (x^{-2})^3 = x^{-2 \times 3} = x^{-6} = \frac{1}{x^6} \]

To raise a product to a power, multiply the power by the exponent of each base. (Power of a product
   = Product of the powers)

\[ (ab)^2 = a^2b^2 \quad 5(ab)^2 = 5a^2b^2 \]
V. Dividing Powers: To divide powers with the same base, keep the base and subtract the exponents.

\[
\frac{a^3}{a} = a^{3-1} = a^2 \quad \frac{b^2}{b^5} = b^{2-5} = b^{-3} = \frac{1}{b^3} \quad \frac{c^0}{c^{-3}} = c^{0-(-3)} = c^3
\]

In the alternative, with positive exponents, you can compare the exponents in the numerator and denominator, and cancel the common factors.

\[
\frac{a^3}{a} = a^{3-1} = a^2 \quad \frac{a^3}{a^2} = a
\]

Multiplying polynomials: multiply the coefficients separately, then multiply each variable separately.

VI. Multiplying a monomial by a polynomial: multiply the monomial by each term of the polynomial in order.

E.g. \(3ab(2a^2 - 4b + c) = 6a^3b - 12ab^2 + 3abc\)

VII. Multiplying a binomial by a polynomial: multiply the first term of the binomial by the first term of the polynomial, then multiply the first term of the binomial by the second term of the polynomial, and continue until the first term of the binomial has been multiplied with each term of the polynomial; then multiply the second term of the binomial with each term of the polynomial; then combine like terms to simplify the answer.

E.g. \((2a - 3b)(a + 2b - 3c) = 2a^2 + 4ab - 6ac - 3ab - 6b^2 + 9bc = 2a^2 + ab - 6ac - 6b^2 + 9bc\) (ab’s are like terms)

A mnemonic device to remember the order for multiplying two binomials is FOIL (first, outer, inner, last)

E.g. \((x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9\)

• No shortcuts: \((x - 3)^2 \neq x^2 - 9\), 
  \(\neq x^2 + 9\) \((x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9\) (don’t lose the middle term)

  Also \((a + b)^3 \neq a^3 + b^3\)
Adding, Subtracting, Multiplying, and Simplifying Radicals

\( \sqrt{2} \) is an irrational number that can be represented exactly only by using the radical sign; It can only be approximated using a decimal (e.g. 1.414).

\( \sqrt{ } \) refers to only the principal or positive square root of the radicand E.g. \( \sqrt{9} = 3 \), however, if \( x^2 = 9 \), \( x = \pm \sqrt{9} \), or \( \pm 3 \)

I. Simplifying square roots or radicals:

To simplify a square root, factor the radicand into perfect squares and non-perfect squares, and then simplify the perfect square part of the radicand.

E.g. \( \sqrt{48} = \sqrt{16*3} = \sqrt{16} * \sqrt{3} = 4\sqrt{3} \)

Binomial radicands must be treated as a single term under the radical.

E.g. \( \sqrt{25-16} \neq \sqrt{25} - \sqrt{16} \) E.g. \( \sqrt{x^2+9} \neq x+3 \) E.g. \( \sqrt{-x^2+9} \neq -\sqrt{x^2-9} \),

however, if you can factor the binomial: e.g. \( \sqrt{12x^2-16x^2} = \sqrt{4x^2(3x-4)} = 2x\sqrt{3x-4} \)

You should equate \( \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \) and you should reduce \( \frac{2}{\sqrt{2}} \) to \( \sqrt{2} \), not convert it to \( \frac{2\sqrt{2}}{2} \).

II. Adding and subtracting square roots or radicals: Handle the radical as you would handle a variable.

To add or subtract radicals, the radicals must be “alike”. \( 2 + \sqrt{2} \) cannot be simplified; however, \( 3 + \sqrt{9} = 3 + 3 = 6 \).

\[ 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3} \] (same as \( 2x + 3x = 5x \))

\[ 2\sqrt{3} + 3\sqrt{2} \] cannot be simplified. (\( \sqrt{3} \)'s cannot be added to \( \sqrt{2} \)'s and simplified)

Sometimes radicals that do not appear alike can be added or subtracted if the radicals are simplified.

E.g. \( \sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3} \)

III. Multiplying square roots or radicals:

To multiply radicals, multiply the radicands and then try to factor a perfect square in the product radicand.

E.g. \( \sqrt{2} * \sqrt{3} = \sqrt{6} \) \( \sqrt{2} * \sqrt{6} = \sqrt{12} = 2\sqrt{3} \) \( \sqrt{10} * \sqrt{15} = \sqrt{150} = \sqrt{25*6} = 5\sqrt{6} \)

When multiplying radicals with whole number coefficients, handle them like variables:
multiply the coefficients separately, then multiply the radicands, and simplify if possible.

\[ 3\sqrt{2} * 2\sqrt{3} = 6\sqrt{6} \]
\[ 3\sqrt{6} * 2\sqrt{10} = 6\sqrt{60} = 6\sqrt{4*15} = 6\sqrt{4} \sqrt{15} = 12\sqrt{15} \]
IV. Squaring a square root or radical:

\[ \sqrt{2^2} = \sqrt{2^2} = 2 \]

V. Squaring a fraction: To square a fraction, square the numerator and denominator separately.

\[ \left( \frac{1}{2} \right)^2 = \frac{1^2}{2^2} = \frac{1}{4} \]

VI. Taking the square root of a fraction: If possible, simplify the fraction first; then take the square root of the numerator and denominator separately.

E.g. \[ \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \quad \frac{\sqrt{9}}{\sqrt{12}} = \frac{3}{\sqrt{4}} = \frac{3}{2} \]

The square root (or any even root) of a negative number results in a complex number with \( i \) as part of the answer.

E.g. \( \sqrt{-1} = i \quad \sqrt{-8} = \sqrt{(-1)(8)} = \sqrt{-1}\sqrt{8} = i \cdot 2\sqrt{2} = 2i\sqrt{2} \)

The cube root (or any odd root) of a negative number results in a real, non-imaginary number.

E.g. \( \sqrt[3]{-1} = -1 \quad \sqrt[3]{-8} = -2 \)

When solving an equation with a radical, you cannot square the numerator and denominator of a fraction without squaring both sides of the equation; and if you do, you must check for extraneous solutions.

E.g. Solve: \( \frac{x}{\sqrt{2}} = 3 \) Incorrect: \( \frac{x^2}{2} = 3 \) Correct: \( \frac{x^2}{2} = 9 \) Solving: \( x^2 = 18 \)

\[ x = \pm 3\sqrt{2} \text{ but } -3\sqrt{2} \text{ must be rejected as an extraneous solution created by the act of squaring both sides of the equation.} \]

Squaring the numerator and denominator of a fraction changes its value. E.g. \( \frac{2}{3} \neq \frac{2^2}{3^2} \)

Form of answers:
If an **exact answer** is requested, leave irrational numbers, such as \( \sqrt{2}, \sqrt{3}, \pi, \text{ or } e \), in the answer. Do not give decimal approximations.

A fraction should not contain a decimal or another fraction in either the numerator or denominator.

A.P. Calculus does not require rationalization of denominators.

A.P. Calculus requires 3 decimal places for inexact final answers. Round inexact intermediate steps to at least 6 decimal places to insure 3 decimal accuracy in the final answer.
§ 1.1 Lines, linear equations, systems of linear equations, intersections, distance, midpoint

Any 2 points determine a line (only one line can pass through 2 points) – Euclid’s 1st Postulate (paraphrased)

From 2 points, you can calculate the slope of a line, and use the slope and one point to write a linear equation in point-slope form: \( y - y_1 = m(x - x_1) \) – the most useful form for calculus

If you know the y-intercept, you can write the linear equation in slope-intercept form: \( y = mx + b \)

You can also write a linear equation in standard or general form: \( Ax + By = C \) – useful for representing data as follows: 2 CDs and 3 DVDs cost $56.75 \( \rightarrow \) 2c + 3d = 56.75

Graphing linear equations: (1) plot y-intercept and use slope to move up/down to another point (2) plot the x and y-intercepts (How do we find the x or y-intercept?)

Horizontal lines: same y value for all points on the horizontal line; equation: \( y = \# \); slope = 0

Vertical lines: same x value for all points on the line; equation: \( x = \# \); slope = no slope/undefined

Intersection of lines: useful for determining breakeven points (economic analysis)

(1) determine intersection of graphs
(2) solve a system of linear equations algebraically (solve simultaneous equations or solve the equations simultaneously)

Methods for solving a system of linear equations algebraically: (1) substitution
(2) elimination (aka linear combination in McDougal – Littell)

Possible solutions: (1) one solution, e.g. \( x = \# \), \( y = \# \); intersection in 1 point geometrically
(2) no solution – inconsistent system, e.g. \( 0 = 2 \); parallel lines (graphs don’t intersect)
(3) infinite # of solutions – dependent system, e.g. \( 3 = 3 \); all points on the same line (“coinciding lines” in Brown)

(3) find intersections on calculator using 2nd Trace: Calc: 5 - intersection

Distance (length of a segment AB) \( AB = \text{distance or length from endpoint A to endpoint B} \)

Use Pythagorean Theorem

Distance formula: \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

Midpoint of a line segment: \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \) – average x-values of the endpoints, average y-values of the endpoints
§ 1.2 Slopes, parallel, perpendicular lines

Slope of a line from 2 points: \[
\frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}
\]

Parallel lines: same slope  \quad \text{Perpendicular lines: slopes are opposite reciprocals (product = -1)}

§ 1.3 Forms of linear equations: point-slope \( y - y_1 = m (x - x_1) \); slope-intercept \( y = mx + b \); standard or general \( Ax + By = C \), where \( A, B, \) and \( C \) are integers, and \( A \geq 0 \)

Perpendicular bisector: need midpoint and perpendicular slope, then use point slope form to write a linear equation

§ 1.4 Linear functions and models

Relations describe a connection between an input quantity (domain) and one or more output values (range)

A function describes a dependent relationship between an input quantity and only one output value

Function notation: \( F(x) \) or \( f(x) \) "f of x" describes the function (operation) \( f \) with \( x \) as its input quantity

\[ f(x) = \quad \text{is equivalent to} \quad y = \quad \]

Step function

Domain v. Range

<table>
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<th>Range</th>
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<tbody>
<tr>
<td>Input</td>
<td>Output</td>
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<tr>
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<td>Dependent variable</td>
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<td>( y )</td>
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<td>2nd letter alphabetically if letters are chosen randomly</td>
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<td>1st number of an ordered pair (( _, ) )</td>
<td>2nd number of an ordered pair (( _, ) )</td>
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Time is usually the independent variable

The independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.
Linear Equations Worksheet #1

1. Do A – G, for each of the following pairs of points: \((-3, 4) (2, 1); (-3, -5) (3, -1); (2, 5) (6, 3)\)
   A) Find the slope of the line through those points
   B) Use the slope and one point to write a linear equation in point-slope form: \(y - y_1 = m(x - x_1)\)
   C) Convert point-slope form to slope-intercept form: \(y = mx + b\)
   D) Use the slope from (A) and one point to write a linear equation in slope-intercept form
   E) Convert slope-intercept form to standard form: \(Ax + By = C\)
   F) Find the x-intercept of the linear equation in coordinate form \((__, __)\)
   G) Find the y-intercept of the linear equation in coordinate form \((__, __)\)

H) Generate the equation for a vertical line through the point \((-2, 1)\)
I) Generate the equation for a horizontal line through the point \((-2, 1)\)

Given the linear equation \(y = \frac{-3}{5}x - \frac{11}{5}\)

J) Generate the equation for the parallel line through the point \((-2, -1)\)
K) Generate the equation for the perpendicular line through the point \((-2, -1)\)

Given the linear equation \(y = \frac{2}{3}x + 3\)

L) Generate the equation for the parallel line through the point \((2, 1)\)
M) Generate the equation for the perpendicular line through the point \((2, 1)\)

Given the linear equation \(y = -\frac{1}{2}x - \frac{2}{3}\)

N) Generate the equation for the parallel line through the point \((1, 2)\)
P) Generate the equation for the perpendicular line through the point \((1, 2)\)
2. Find the length of and the coordinates of the midpoint of $\overline{CD}$:
   (a) $C(1, 0)$ and $D(7, 8)$
   (b) $C(3, 3)$ and $D(15, 12)$
   (c) $(-8, -3)$ and $(7, 5)$
   (d) $C\left(\frac{7}{2}, -1\right)$ and $D\left(-\frac{5}{2}, \frac{7}{2}\right)$

3. Graph $3x - 2y = 6$. Label the $x$ and $y$-intercepts with their coordinates. Find the area of the triangle created by the origin and the $x$ and $y$-intercepts.

4. Solve each pair of equations simultaneously using either the substitution or the elimination method.
   (a) $2x + 3y = 15$
   $4x - 9y = 3$
   (b) $-2x - 6y = 18$
   $x - 3y = 6$
   (c) $x - 3y = 4$
   $5x + y = -8$

5. Find the slope of the line joining the following pairs of points:
   (a) $(-7, 5)$, $(7, 5)$
   (b) $(-3, 2)$, $(-3, -2)$
   (c) $(a, b)$, $(b, a)$
   (d) $(a, b)$, $(b, a)$
   (e) $(a, b)$, $(b, a)$

6. Find the slope and $x$ and $y$-intercepts (in coordinate form) for the following lines:
   (a) $3x + 9y = 7$
   (b) $4y = 11x$
   (c) $y = -2$
   (d) $x = 6$

7. Write a linear equation for each line:
   (a) the line with $x$-intercept $(-2, 0)$ and $y$-intercept $(0, 4)$
   (b) the line with $x$-intercept $(6, 0)$ and parallel to the line $5x + 4y = 1$
   (c) the line through $(8, -2)$ and perpendicular to the line $y = 7 - 2x$
   (d) the line through $(8, -2)$ and parallel to $y = 4$
   (e) the line through $(8, -2)$ and perpendicular to $y = 4$

8(a). Write a linear equation for the perpendicular bisector of the segment joining $(0, 3)$ and $(-4, 5)$
8(b). Write a linear equation for the perpendicular bisector of the segment joining $(2, 4)$ and $(4, -4)$

9(a). If $f(x) = 5x - 10$, find $f(-3)$.
9(b). Find the zero of $f(x)$. 
10(a). If \( g(x) = -1 \), find \( g(-3) \).
10(b). If \( g(x) = -1 \), find the zero(s) of \( g(x) \).

11. The linear function \( S(n) \) has a zero at 3, and intersects the vertical axis at \(-2\).
Write a linear equation for \( S(n) \).

12. The graph of the linear function \( h(t) \) has a slope of \(-2\), and intersects the \( t \)-axis at \( t = 6 \). Write a linear equation for \( h(t) \).

13. If \( f(x) \) is a linear function with \( f(1) = 5 \), and \( f(3) = 9 \), write a linear equation for \( f(x) \).

14. If \( g(x) \) is a linear function with \( g(-1) = -3 \), and \( g(-4) = 12 \), write a linear equation for \( g(x) \).
Sample of Linear Equations Worksheet #1

Given two points: \((2, -3), (-1, 1)\)

A) Find the slope of the line through those points \(\frac{y_2-y_1}{x_2-x_1}\) or \(\frac{y_1-y_2}{x_1-x_2}\)

\[
\begin{align*}
\frac{1-(-3)}{-1-2} &= -\frac{4}{3} \\
\frac{-3-1}{2-(-1)} &= -\frac{4}{3}
\end{align*}
\]

B) Use the slope and one point to write a linear equation

\[y - (-3) = -\frac{4}{3}(x - 2)\] or \[y - 1 = -\frac{4}{3}(x - (-1))\]

in point-slope form \(y - y_1 = m(x - x_1)\)

\[
\begin{align*}
y + 3 &= -\frac{4}{3}x + \frac{8}{3} \\
y - 1 &= -\frac{4}{3}x - \frac{4}{3}
\end{align*}
\]

\[
\begin{align*}
-3 &= -\frac{9}{3} + 1 \\
+1 &= +\frac{3}{3}
\end{align*}
\]

C) Convert point-slope form to slope-intercept form \((y = mx + b)\)

\[
\begin{align*}
y &= -\frac{4}{3}x + b \\
y &= -\frac{4}{3}x - \frac{1}{3}
\end{align*}
\]

D) Use the slope and one point to write a linear equation

in slope-intercept form \((y = mx + b)\)

\[
\begin{align*}
y &= -\frac{4}{3}x + b \\
1 &= -\frac{4}{3}(-1) + b \\
1 &= \frac{4}{3} + b \\
\frac{1}{3} &= b \\
y &= -\frac{4}{3}x - \frac{1}{3}
\end{align*}
\]

E) Convert slope-intercept form to standard form \((Ax + By = C)\)

where \(A \geq 0\) and \(A, B,\) and \(C\) are integers

\[4x + 3y = -1\]

F) Find the x-intercept of the linear equation in coordinate form \((?, 0)\)

\[
\begin{align*}
4x + 3(0) &= -1 \\
x &= -\frac{1}{4} \\
(-\frac{1}{4}, 0)
\end{align*}
\]

G) Find the y-intercept of the linear equation in coordinate form \((0, ?)\)

or read it from slope-intercept form: \(b = y\) coordinate of the y-intercept

\[
\begin{align*}
4(0) + 3y &= -1 \\
3y &= -1 \\
y &= -\frac{1}{3} \\
(0, -\frac{1}{3})
\end{align*}
\]
Given a point: \((-2, 5)\)

H) Generate the equation for a vertical line through the point \((x = x\ coordinate)\)
\[ x = -2 \]

I) Generate the equation for a horizontal line through the point \((y = y\ coordinate)\)
\[ y = 5 \]

Given a linear equation:
\[ y = \frac{-4}{3}x - \frac{1}{3} \]

J) Generate the equation for a parallel line through a specific point \((-2, 5)\)
(parallel lines have the same slopes)
\[ y - 5 = \frac{-4}{3}(x + 2) \]
\[ y - 5 = \frac{-4}{3}x - \frac{8}{3} \]
\[ y = \frac{-4}{3}x + \frac{7}{3} \]

K) Generate the equation for a perpendicular line through a specific point \((-2, 5)\)
(perpendicular lines have slopes that are opposite reciprocals)
(the product of the slopes of perpendicular lines is -1)
\[ y - 5 = \frac{3}{4}(x + 2) \]
\[ y - 5 = \frac{3}{4}x + \frac{3}{2} \]
\[ y = \frac{3}{4}x + \frac{13}{2} \]
Pre-calculus: Solving Quadratic Equations Algebraically: Completing the Square

To solve a quadratic equation algebraically, you can: (1) factor, (2) take square roots of both sides (if there is no linear term), (3) use the quadratic formula (use the discriminant to determine the type and number of solutions), or (4) complete the square.

To complete the square to solve \( ax^2 + bx + c = 0 \)

\[
\frac{ax^2 + bx + c}{a} = 0
\]

\[
x^2 + \frac{b}{a}x + \frac{c}{a} = 0
\]

\[
x^2 + \frac{b}{a}x = -\frac{c}{a}
\]

\[
x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}
\]

\[
\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}
\]

1\(^{st}\): Divide both sides of the equation by the coefficient of \( x^2 \) to produce \( 1x^2 \).

2\(^{nd}\): Move the constant to the right side.

3\(^{rd}\): Take \( \frac{1}{2} \) the coefficient of \( x \), square it and add it to both sides. \( \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} \)

4\(^{th}\): Rewrite left side as a binomial squared

\[
\left(x \pm \frac{b}{2a}\right)^2
\]

5\(^{th}\): Combine the right side

\[
\left(x \pm \frac{1}{2} \frac{b}{a}\right)^2
\]

6\(^{th}\): Take the square root of both sides (remember \( \pm \sqrt{\cdot} \))

7\(^{th}\): Solve for \( x \)

\[
x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}
\]

\[
x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}
\]

\[
x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}
\]
To complete the square to solve $2x^2 + 3x - 20 = 0$

1\textsuperscript{st}: Divide both sides of the equation by 2, the coefficient of $x^2$ to produce $1x^2$.

\[
\frac{2x^2 + 3x - 20}{2} = \frac{0}{2}
\]

\[
x^2 + \frac{3}{2}x - 10 = 0
\]

2\textsuperscript{nd}: Move the constant to the right side.

\[
x^2 + \frac{3}{2}x = 10
\]

3\textsuperscript{rd}: Take $\frac{1}{2}$ the coefficient of $x$, square it and add it to both sides. $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

\[
x^2 + \frac{3}{2}x + \frac{9}{16} = 10 + \frac{9}{16}
\]

4\textsuperscript{th}: Rewrite left side as a binomial squared

\[
\left(x + \frac{3}{4}\right)^2 = \frac{169}{16}
\]

5\textsuperscript{th}: Combine the right side

\[
\left(x + \frac{3}{4}\right)^2 = \frac{160}{16} + \frac{9}{16}
\]

6\textsuperscript{th}: Take the square root of both sides (remember $\pm\sqrt{\ }$)

\[
x + \frac{3}{4} = \pm \sqrt{\frac{169}{16}}
\]

7\textsuperscript{th}: Solve for $x$ by isolating $x$

\[
x = -\frac{3}{4} \pm \sqrt{\frac{169}{16}}
\]

8\textsuperscript{th}: If possible, simplify

\[
x = -\frac{3}{4} \pm \frac{13}{4} = -\frac{-3+13}{4}, -\frac{-3-13}{4} = \frac{5}{2}, -4
\]
To complete the square to solve $x^2 + 5x + 2 = 0$

1\textsuperscript{st}: Skip step; already in form of $1x^2$.

2\textsuperscript{nd}: Move the constant to the right side.

3\textsuperscript{rd}: Take $\frac{1}{2}$ the coefficient of $x$, square it and add it to both sides. $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

4\textsuperscript{th}: Rewrite left side as a binomial squared $\left(x \pm \frac{1}{2} b\right)^2$, i.e. $\left(x + \frac{1}{2} \cdot 5\right)^2$

5\textsuperscript{th}: Combine the right side

6\textsuperscript{th}: Take the square root of both sides (remember $\pm \sqrt{\cdot}$)

7\textsuperscript{th}: Solve for $x$ by isolating $x$.

\[ x + \frac{5}{2} = \pm \sqrt{\frac{17}{4}} \]

\[ x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2} \text{ or } -5 + \frac{\sqrt{17}}{2} \]

---

To complete the square to solve $x^2 - 8x + 20 = 0$

1\textsuperscript{st}: Skip step; already in form of $1x^2$.

2\textsuperscript{nd}: Move the constant to the right side.

3\textsuperscript{rd}: Take $\frac{1}{2}$ the coefficient of $x$, square it and add it to both sides. $(-4)^2 = 16$

4\textsuperscript{th}: Rewrite left side as a binomial squared $\left(x \pm \frac{1}{2} b\right)^2$, i.e. $\left(x - \frac{1}{2} \cdot 8\right)^2$

5\textsuperscript{th}: Combine the right side

6\textsuperscript{th}: Take the square root of both sides (remember $\pm \sqrt{\cdot}$)

7\textsuperscript{th}: Solve for $x$ and simplify.

\[ x - 4 = \pm \sqrt{-4} \]

\[ x = 4 \pm 2i \]
Solving Quadratic Equations Worksheet

1. Solve the equations by factoring (remember the prerequisites: set = 0, and GCF):

(a) \(2x^2 - 50 = 0\)  
(b) \(9x^2 + 6x = 0\)  
(c) \(3x^2 + 2x = 8\)

(d) \(2x^2 + 18x + 36 = 0\)  
(e) \(6x^2 - 5x - 6 = 0\)  
(f) \(6x^2 + 5x - 6 = 0\)

(g) \(9x^2 + 16x - 4 = 0\)  
(h) \(12x^2 - 9x - 3 = 0\)  
(i) \(3x^2 - 12x + 12 = 0\)

2. Solve the equations by taking square roots of both sides:

(a) \(2x^2 = 36\)  
(b) \(3x^2 = 27\)  
(c) \(2x^2 - 7 = x^2 + 13\)  
(d) \(x^2 = -12\)

(e) \((2x - 1)^2 = -4\)  
(f) \((3x + 4)^2 = 144\)
3. Solve the equations by using the quadratic formula 
\( x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} \)  
(a) \( x^2 + x - 1 = 0 \)  
(b) \( x^2 - 4x - 8 = 0 \)  
(c) \( 2x^2 + 7 = 4x \)  
(d) \( 3x^2 + 6x + 9 = 0 \)  
(e) \( 3x^2 + 3x = 2 \)  
(f) \( 10x^2 + 4x - 2 = 0 \)

4. Solve the equations by completing the square.  
(a) \( x^2 + 6x + 10 = 0 \)  
(b) \( x^2 - 8x = 2 \)  
(c) \( 2x^2 + 3x - 4 = 0 \)
§ 1.5 Complex numbers, real + imaginary parts, simplifying, operations, conjugates

Using the real number system, one cannot take the square root (or any even root, e.g. the fourth root, $\sqrt[4]{}$) of a negative number.

Using the real number system, it is possible to take the cube root ($\sqrt[3]{-1}$), or any odd root, of a negative number.

E.g. $\sqrt[3]{-1} = -1$ because $(-1)(-1)(-1) = -1$; but $? \times ? = -1$

To use square roots of negative numbers, the imaginary number i was invented. i was defined as $\sqrt{-1}$

The imaginary number cycle:

\[ i = \sqrt{-1}, \quad i^2 = \sqrt{-1} \times \sqrt{-1} = -1, \quad i^3 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1}, \quad i^4 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = 1 \]

\[ (-1 \times \sqrt{-1} = -i, \quad (-1) \times (-1) = 1) \]

this cycle repeats every four i; e.g. $i^5 = 1 = i$; $i^{10} = 2 = -1$; $i^{19} = 3 = i$; $i^{36} = 4 = 1$

Complex numbers combine real and imaginary numbers. Complex numbers are written in a + bi form, where a = the real part, and bi = the imaginary part.

When adding or subtracting complex numbers, add or subtract the real parts, then add or subtract the imaginary parts; then recombine in a + bi form.

E.g. $(3 + i) - (2 - 3i) = 1 + 4i$

When multiplying complex numbers, use the distributive property (FOIL), then + / − the real parts, then + / − the imaginary parts; and then recombine in a + bi form.

E.g. $(3 + 4i) (5 + 2i) = 15 + 3(2i) + 5(4i) + (4i) (2i) = 15 + 6i + 20i + 8i^2 = 15 + 26i + (-8) = 7 + 26i$

note that $i^2 = -1$

When dividing complex numbers, or simplifying a fraction with a complex denominator, multiply numerator and denominator by the conjugate of the denominator; this will convert the denominator to a real number. a + bi and a − bi are conjugates.

E.g. $\frac{1-i}{1+i} = \frac{1-i}{1+i} \times \frac{1-i}{1-i} = \frac{1-i-i+i^2}{1-i+i^2} = \frac{1-2i-1}{1+1} = \frac{-2i}{2} = -i$

note that $i^2 = -1$

The sum or product of complex conjugates are real numbers.
Complex Numbers Worksheet

1. Simplify each expression:
   (a) \( i^7 = \) \( i^{13} = \) \( i^{22} = \) \( i^{48} = \) \( i^{-3} = \)

   (f) \( \sqrt{-2} \times \sqrt{-5} = \)

   (g) \( \frac{\sqrt{-25}}{\sqrt{-50}} = \)

   (h) \( (7 - 8i) - (6 + 2i) = \)

   (i) \( \frac{1}{8}(7 - 2i) + \frac{3}{8}(5 - 5i) = \)

   (j) \( (7 + 3i)(7 - 3i) = \)

   (k) \( (\sqrt{3} + 4\sqrt{2}i)(\sqrt{3} - 4\sqrt{2}i) = \)

   (l) \( (5 - 2i)(-1 + 3i) = \)

   (m) \( (4 + 7i)^2 = \)

   (n) \( |2 - 3i| = \)

2. Perform the division by multiplying the numerator and denominator by the conjugate of the denominator, and write your simplified answer in \( a + bi \) form.

   (a) \( \frac{1}{4 - 3i} = \)

   (b) \( \frac{3 - 2i}{3 + 2i} = \)

   (c) \( \frac{2 + \sqrt{5}i}{3 - \sqrt{5}i} = \)
Pre-calculus – Proper Notation

1. Coefficients always precede variables.
   
   E.g. Two times a variable \( x \) is represented as \( 2x \) or \( 2(x) \), but not \( x2 \), or \( (x)2 \), or \( x(2) \).

2. Do not use the symbol \( x \) for multiplication if you are using any variables in an expression.
   
   E.g. Does \( 2 \times x \times x \) mean: two multiplied by the variable \( x \) (2 times \( x \))? or 
   
   two multiplied by the variable \( x \) multiplied by the variable \( x \) (2 times \( x \times x \))?

   Does \( 2 \times y \times y \) mean: two multiplied by the variable \( y \) (2 times \( y \))? or 
   
   two multiplied by the variable \( x \) multiplied by the variable \( y \) (2 times \( x \times y \))? 

   An asterisk (*) is preferable to an \( x \) as a multiplication symbol. Also see # 4 below.

3. Do not use the dot symbol (\( \cdot \)) for multiplication if you are using decimals in the expression.
   
   E.g. Do not write three multiplied by one-half (three times one-half) as \( 3 \cdot 0.5 \).

4. Become comfortable with expressing the multiplication operation by using parentheses or merely placing a number and a variable adjacent to each other with no explicit operation symbol.

   Parentheses are a useful means of avoiding the \( x \) or dot (\( \cdot \)) symbols for multiplication, as well as representing a fraction.

   E.g. \( 3 \cdot (0.5) \) is repetitive. \( 3(0.5) \) is sufficient.
   
   E.g. \( \frac{1}{3} \neq \frac{1}{3} \), \( \frac{1}{3} = \frac{1}{3} \), \( (\frac{1}{3})^2 = \frac{1}{9} \)

5. Distinguish between the positions (and purposes) of exponents and subscripts.

   E.g. \( x^2 \) means \( x \) multiplied by \( x \), or \( x \) squared. \( x_2 \) means the second variable labeled as \( x \).

   E.g. \( f^{-1}(x) \) read as “\( f \) inverse of \( x \)” refers to the inverse of the function \( f(x) \)

   \( x^{-1} \) and \( 2^{-1} \) refer to the reciprocals of \( x \) and \( 2 \), namely \( \frac{1}{x} \) and \( \frac{1}{2} \)

   Distinguish between the \( -1 \) affecting the function notation versus a variable or number.

6. A fraction should not contain a decimal or another fraction in either the numerator or denominator. A fraction is a ratio of integers.

   E.g. \( \frac{0.5}{2} \) should be written as \( \frac{1}{4} \). E.g. \( \frac{\frac{2}{3}}{4} \) should be simplified to \( \frac{1}{6} \).

7. A radicand (the number under the radical symbol (\( \sqrt{\,} \)) should not contain a decimal.

   E.g. \( \sqrt{\frac{3.5}{2}} \) should be written as \( \sqrt{\frac{7}{4}} \) which can be simplified to \( \frac{\sqrt{7}}{2} \).
Graphing Quadratic Functions Worksheet

1. Graph the following quadratic functions. Label the axis of symmetry with its equation. Label the vertex, any x-intercepts, and the y-intercept and its mirror image point, with their coordinates.

(a) \( y = (x - 4)(x + 2) \)

(b) \( y = x^2 + 3x - 10 \)

(c) \( y = 9 - x^2 \)

(d) \( y = -2(x + 3)^2 + 8 \)
2. Convert the following quadratic equations:

(a) \( y = x^2 - 2x + 3 \) to vertex form
(b) \( y = 3(x - 4)^2 + 5 \) to standard form
(c) \( y = (2x - 3)(2x + 3) \) to standard or vertex form

3. Find the coordinates of the intersection points of:

(a) \( y = x + 3, \ y = 4x - x^2 \)
(b) \( x + y = -6, \ y = x^2 + 6x \)